

# X272/13/01

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NATIONAL  
QUALIFICATIONS  
2014

THURSDAY, 22 MAY  
1.00 PM – 3.30 PM

PHYSICS  
ADVANCED HIGHER  
(Revised)

Reference may be made to the Physics Data Booklet and the accompanying Relationships Sheet.

Answer **all** questions.

Any necessary data may be found in the Data Sheet on *Page two*.

Care should be taken to give an appropriate number of significant figures in the final answers to calculations.

Square-ruled paper (if used) should be placed inside the front cover of the answer book for return to the Scottish Qualifications Authority.



**DATA SHEET**  
COMMON PHYSICAL QUANTITIES

Quantity	Symbol	Value	Quantity	Symbol	Value
Gravitational acceleration on Earth	$g$	$9.8 \text{ m s}^{-2}$	Mass of electron	$m_e$	$9.11 \times 10^{-31} \text{ kg}$
Radius of Earth	$R_E$	$6.4 \times 10^6 \text{ m}$	Charge on electron	$e$	$-1.60 \times 10^{-19} \text{ C}$
Mass of Earth	$M_E$	$6.0 \times 10^{24} \text{ kg}$	Mass of neutron	$m_n$	$1.675 \times 10^{-27} \text{ kg}$
Mass of Moon	$M_M$	$7.3 \times 10^{22} \text{ kg}$	Mass of proton	$m_p$	$1.673 \times 10^{-27} \text{ kg}$
Radius of Moon	$R_M$	$1.7 \times 10^6 \text{ m}$	Mass of alpha particle	$m_\alpha$	$6.645 \times 10^{-27} \text{ kg}$
Mean Radius of Moon Orbit		$3.84 \times 10^8 \text{ m}$	Charge on alpha particle		$3.20 \times 10^{-19} \text{ C}$
Solar radius		$6.955 \times 10^8 \text{ m}$	Planck's constant	$h$	$6.63 \times 10^{-34} \text{ J s}$
Mass of Sun		$2.0 \times 10^{30} \text{ kg}$	Permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12} \text{ F m}^{-1}$
1 AU		$1.5 \times 10^{11} \text{ m}$	Permeability of free space	$\mu_0$	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Stefan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$	Speed of light in vacuum	$c$	$3.0 \times 10^8 \text{ m s}^{-1}$
Universal constant of gravitation	$G$	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	Speed of sound in air	$v$	$3.4 \times 10^2 \text{ m s}^{-1}$

**REFRACTIVE INDICES**

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	Refractive index	Substance	Refractive index
Diamond	2.42	Glycerol	1.47
Glass	1.51	Water	1.33
Ice	1.31	Air	1.00
Perspex	1.49	Magnesium Fluoride	1.38

**SPECTRAL LINES**

Element	Wavelength/nm	Colour	Element	Wavelength/nm	Colour
Hydrogen	656	Red	Cadmium	644	Red
	486	Blue-green		509	Green
	434	Blue-violet		480	Blue
	410	Violet	<i>Lasers</i>		
	397	Ultraviolet	<i>Element</i>	<i>Wavelength/nm</i>	<i>Colour</i>
	389	Ultraviolet	Carbon dioxide	9550 } 10590 }	Infrared
Sodium	589	Yellow	Helium-neon	633	Red

**PROPERTIES OF SELECTED MATERIALS**

Substance	Density/ $\text{kg m}^{-3}$	Melting Point/ K	Boiling Point/K	Specific Heat Capacity/ $\text{J kg}^{-1} \text{ K}^{-1}$	Specific Latent Heat of Fusion/ $\text{J kg}^{-1}$	Specific Latent Heat of Vaporisation/ $\text{J kg}^{-1}$
Aluminium	$2.70 \times 10^3$	933	2623	$9.02 \times 10^2$	$3.95 \times 10^5$	....
Copper	$8.96 \times 10^3$	1357	2853	$3.86 \times 10^2$	$2.05 \times 10^5$	....
Glass	$2.60 \times 10^3$	1400	....	$6.70 \times 10^2$	....	....
Ice	$9.20 \times 10^2$	273	....	$2.10 \times 10^3$	$3.34 \times 10^5$	....
Glycerol	$1.26 \times 10^3$	291	563	$2.43 \times 10^3$	$1.81 \times 10^5$	$8.30 \times 10^5$
Methanol	$7.91 \times 10^2$	175	338	$2.52 \times 10^3$	$9.9 \times 10^4$	$1.12 \times 10^6$
Sea Water	$1.02 \times 10^3$	264	377	$3.93 \times 10^3$	....	....
Water	$1.00 \times 10^3$	273	373	$4.19 \times 10^3$	$3.34 \times 10^5$	$2.26 \times 10^6$
Air	1.29	....	....	....	....	....
Hydrogen	$9.0 \times 10^{-2}$	14	20	$1.43 \times 10^4$	....	$4.50 \times 10^5$
Nitrogen	1.25	63	77	$1.04 \times 10^3$	....	$2.00 \times 10^5$
Oxygen	1.43	55	90	$9.18 \times 10^2$	....	$2.40 \times 10^4$

The gas densities refer to a temperature of 273 K and a pressure of  $1.01 \times 10^5 \text{ Pa}$ .

1. The acceleration of a particle moving in a straight line is described by the expression

$$a = 1.2t.$$

At time,  $t = 0$  s the displacement of the particle is 0 m and its velocity is  $1.4 \text{ m s}^{-1}$ .

- (a) Show that the velocity of the particle at time  $t$  is given by the expression

$$v = 0.6t^2 + 1.4.$$

2

- (b) Calculate the displacement of the particle when its velocity is  $3.8 \text{ m s}^{-1}$ .

3

**(5)**

2. A student uses two methods to calculate the moment of inertia of a cylinder about its central axis.

- (a) In the first method the student measures the mass of the cylinder to be  $0.115 \text{ kg}$  and the diameter to be  $0.030 \text{ m}$ .

Calculate the moment of inertia of the cylinder.

2

- (b) In a second method the student allows the cylinder to roll down a slope and measures the final speed at the bottom of the slope to be  $1.6 \text{ m s}^{-1}$ . The cylinder has a diameter of  $0.030 \text{ m}$  and the slope has a height of  $0.25 \text{ m}$ , as shown in Figure 2.

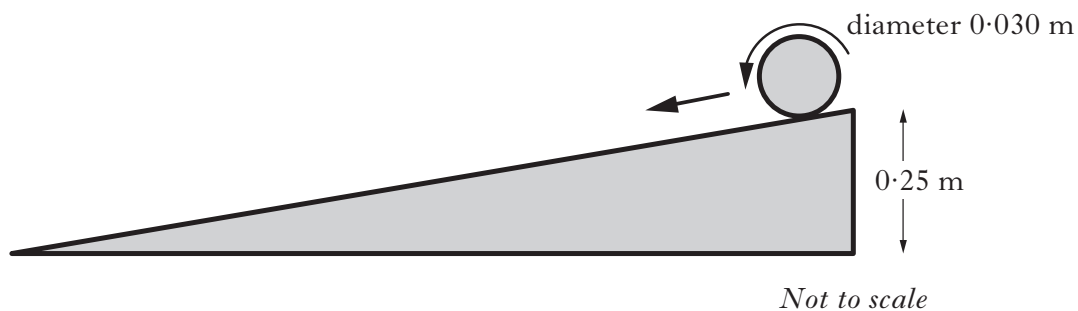


Figure 2

Using the conservation of energy, calculate the moment of inertia.

4

- (c) Explain why the moment of inertia found in part (b) is greater than in part (a).

1

**(7)**

**[Turn over**

3. A team of astrophysicists from a Scottish University has discovered, orbiting a nearby star, an exoplanet with the same mass as Earth.

By considering the escape velocity of the exoplanet, the composition of its atmosphere can be predicted.

- (a) (i) Explain the term *escape velocity*. 1
- (ii) Derive the expression for escape velocity in terms of the exoplanet's mass and radius. 2
- (iii) The radius of this exoplanet is 1.7 times that of the Earth.  
Calculate the escape velocity of the exoplanet. 3
- (b) Astrophysicists consider that a gas will be lost from the atmosphere of a planet if the typical molecular velocity ( $v_{\text{rms}}$ ) is  $\frac{1}{6}$  or more of the escape velocity for that planet.

The table below gives  $v_{\text{rms}}$  for selected gases at 273 K.

<i>Gas</i>	$v_{\text{rms}}$ (m s <sup>-1</sup> )
Hydrogen	1838
Helium	1845
Nitrogen	493
Oxygen	461
Methane	644
Carbon dioxide	393

The atmospheric temperature of this exoplanet is 273 K.

Predict which of these gases could be found in its atmosphere.

2  
**(8)**

**[Turn over for Question 4 on *Page six***

4. Cygnus X-1 is an X-ray source in the constellation Cygnus that astrophysicists believe contains a black hole. An artist's impression is shown in Figure 4A.

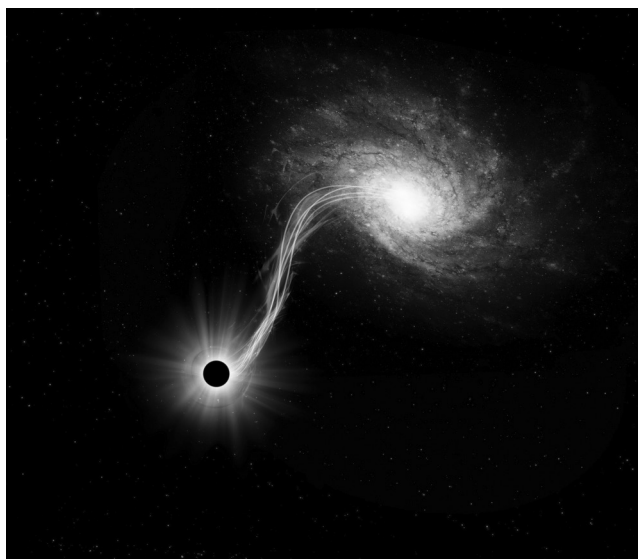


Figure 4A

The mass of the black hole has been determined to be  $14.8$  Solar masses.

- (a) (i) State what is meant by the Schwarzschild radius of a black hole. **1**
- (ii) Calculate the Schwarzschild radius of the black hole in Cygnus X-1. **3**

## 4. (continued)

- (b) The Hertzsprung-Russell (H-R) diagram shown in Figure 4B shows the relationship between the luminosity and surface temperature of stars.

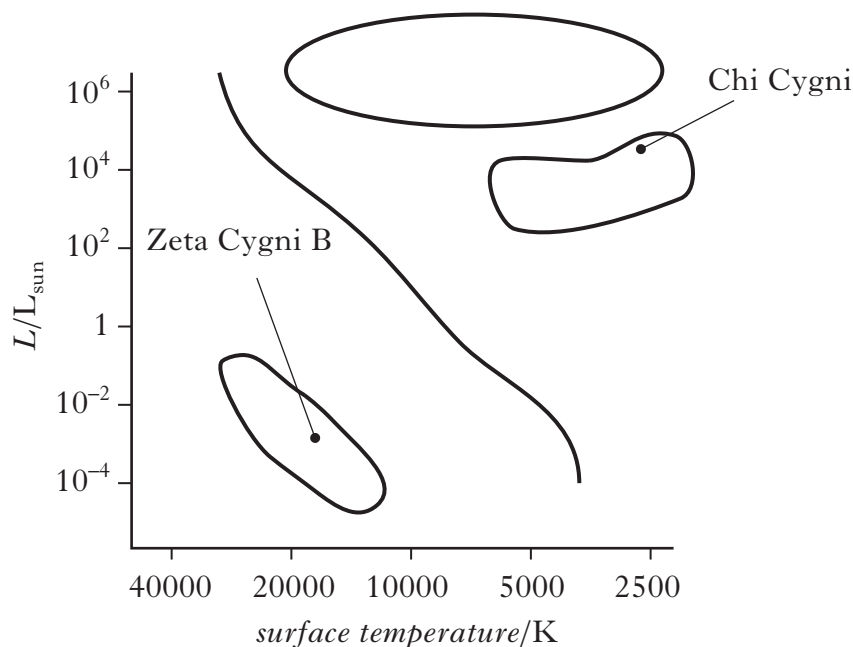


Figure 4B

Zeta Cygni B and Chi Cygni are two stars in the constellation Cygnus. They are shown on the H-R diagram. Chi Cygni is more luminous than Zeta Cygni B.

Describe **two** other differences between these stars.

2

- (c) Another star, Aldebaran B, is a distance of  $6.16 \times 10^{17}$  m from the Earth.

The luminosity of Aldebaran B is  $2.32 \times 10^{25}$  W and its temperature is determined to be  $3.4 \times 10^3$  K.

- (i) Calculate the radius of Aldebaran B.

2

- (ii) Calculate the apparent brightness of Aldebaran B as observed from Earth.

2

(10)

[Turn over

5. A commercial airline pilot talking to his friend, who is a member of the ground crew, states

*“Of course, according to Einstein’s theories, flying at high speed at high altitude means that I’m going to age much slower than you will.”*

Using your knowledge of physics principles, comment on the pilot’s statement. **(3)**

6. In 1928 Davisson and Germer fired a beam of electrons through a very thin layer of nickel in a vacuum, which resulted in the production of a diffraction pattern.

- (a) (i) What did they conclude from the results of their experiment? **1**
- (ii) Give **one** example of experimental evidence that photons of light exhibit particle properties. **1**
- (b) Calculate the de Broglie wavelength of an electron travelling at  $4.4 \times 10^6 \text{ m s}^{-1}$ . **2**
- (c) A 20 g bullet travelling at  $300 \text{ m s}^{-1}$  passes through a 500 mm gap in a target. Using the data given, explain why no diffraction pattern is observed. **2**
- (d) (i) Describe the Bohr model of the hydrogen atom. **2**
- (ii) Calculate the angular momentum of an electron in the third stable orbit of a hydrogen atom. **2**

**(10)**



7. The Sun is constantly losing mass through nuclear fusion. Particles also escape from the corona as shown in Figure 7A. This stream of particles radiating from the Sun is known as the Solar wind and its main constituent, by mass, is protons.



Figure 7A

- (a) Astronomers estimate that the Sun loses mass at a rate of  $1.0 \times 10^9 \text{ kg s}^{-1}$ . This rate has been approximately constant through the Sun's lifetime of  $4.6 \times 10^9$  years.

Estimate the mass lost by the Sun in its lifetime as a percentage of its current mass.

2

- (b) A proton in the solar wind has energy of 3.6 MeV.

(i) Calculate the velocity of this proton.

3

- (ii) The proton enters the magnetic field around the Earth at an angle of  $50^\circ$  as shown in Figure 7B. The magnetic field strength is  $58 \mu\text{T}$ .

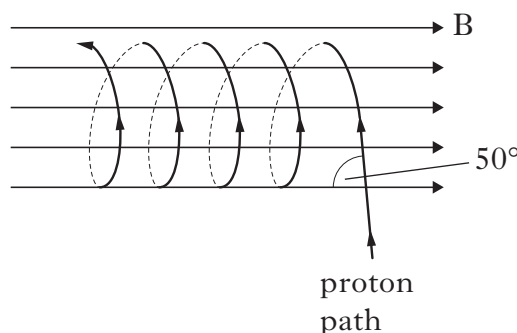


Figure 7B

- (A) Explain the shape of the path followed by the proton in the magnetic field.
- (B) Calculate the radius of curvature of this path.
- (iii) An antiproton of energy 3.6 MeV enters the same region of the Earth's magnetic field at an angle of  $30^\circ$  to the field.

2

3

Describe **two** differences in the paths taken by the antiproton and the original proton.

2

(12)

8. Car engines use the ignition of fuel to release energy which moves the pistons up and down, causing the crankshaft to rotate.

The vertical motion of the piston approximates to simple harmonic motion.

Figure 8 shows different positions of a piston in a car engine.

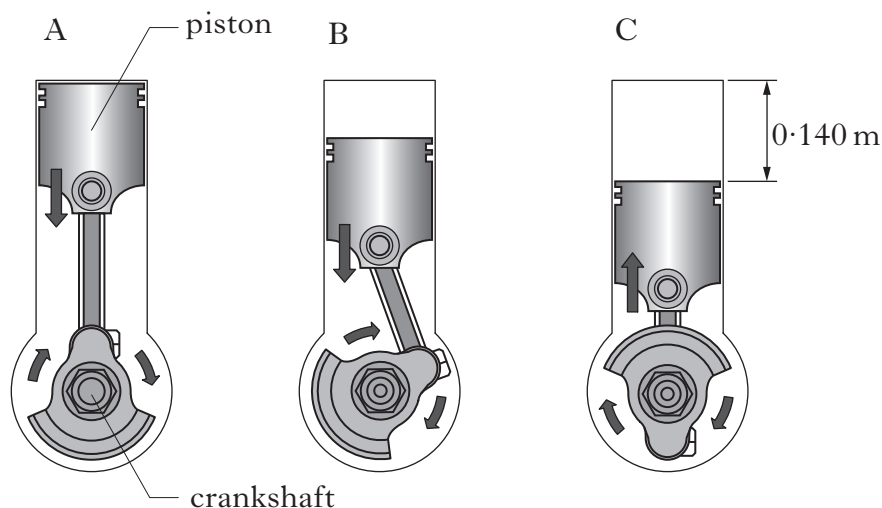


Figure 8

- (a) Define *simple harmonic motion*. 1
- (b) Determine the amplitude of the motion. 1
- (c) In this engine the crankshaft rotates at 1500 revolutions per minute and the piston has a total mass of 1.40 kg.
- (i) Calculate the maximum acceleration of the piston. 3
- (ii) Calculate the maximum kinetic energy of the piston. 2
- (7)**

9. A series of coloured LEDs are used in the Young's slit experiment as shown in Figure 9. The distance from the slits to the screen is  $(2.50 \pm 0.05)\text{m}$ . The slit separation is  $(3.0 \pm 0.1) \times 10^{-4}\text{m}$ .

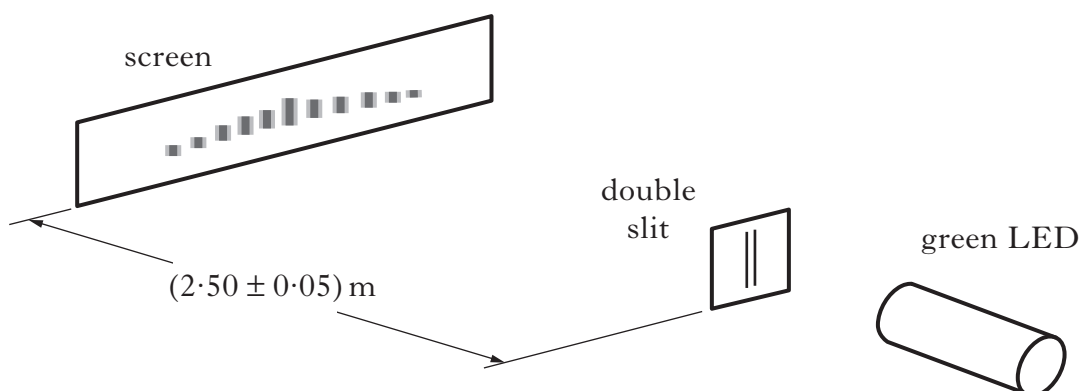


Figure 9

<i>Colour of LED</i>	<i>Wavelength (nm)</i>
Red	$650 \pm 2$
Green	$510 \pm 2$
Blue	$470 \pm 2$

- (a) State whether the pattern on the screen is caused by the division of wavefront or the division of amplitude. 1
- (b) (i) Calculate the fringe separation observed on the screen when the green LED is used. 2
- (ii) Calculate the absolute uncertainty in the fringe separation. 3
- (6)**

**[Turn over**

10. (a) When sunlight hits a thin film of oil floating on the surface of water, a complex pattern of coloured fringes is observed.

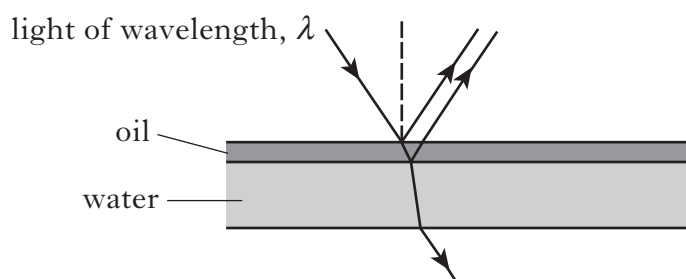


Figure 10

Explain how these fringes are formed.

2

- (b) The surface of a lens is coated with a thin film of magnesium fluoride.

Calculate the minimum thickness required to make the lens non-reflecting at a wavelength of 555 nm.

2

- (c) The lens of a digital camera appears to be purple in white light.

Explain this observation.

2

**(6)**

11. A student, wearing polarising sunglasses, is using a tablet computer outdoors. The orientation of the tablet seems to affect the image observed by the student.

Two orientations are shown in Figure 11A.

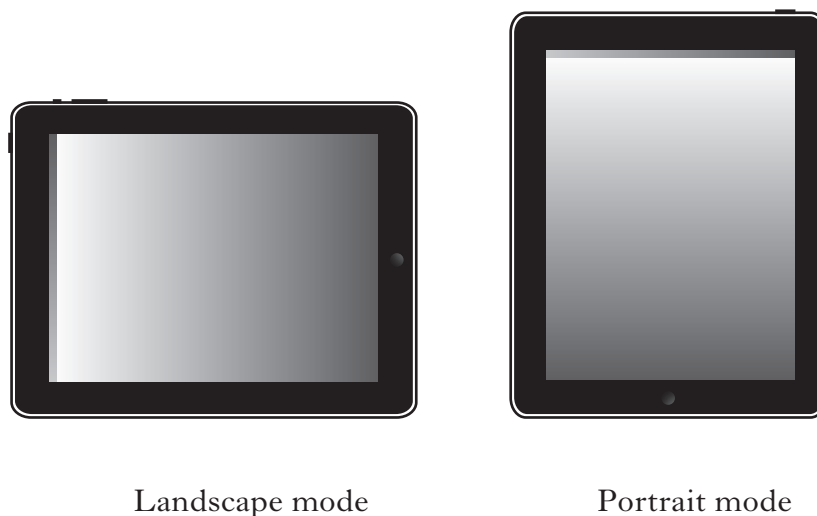


Figure 11A

- (a) In landscape mode the image appears bright and in portrait mode it appears dark.

- (i) What may be concluded about the light emitted from the tablet screen? 1
- (ii) The student slowly rotates the tablet. Describe the change in brightness observed by the student as it is rotated through  $180^\circ$ . 2

- (b) Unpolarised sunlight is incident on a water surface as shown in Figure 11B.

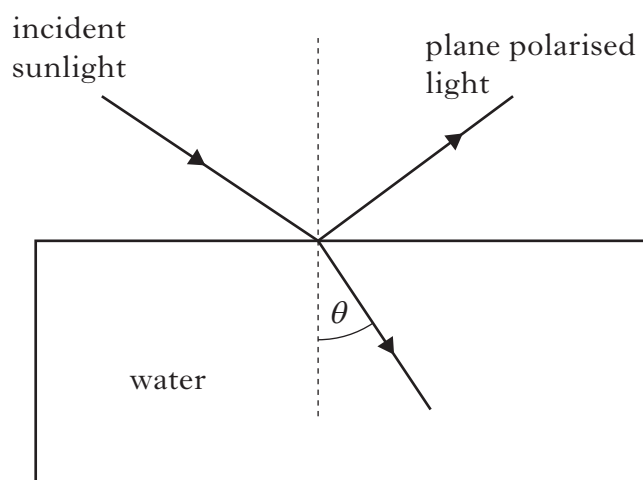


Figure 11B

The light is 100% plane polarised on reflection.

Calculate the angle of refraction  $\theta$ .

2  
(5)

12. Two students conduct a series of experiments to investigate electric and magnetic fields.

The students measure the electric field around a positive point charge  $Q$ . They measure the electric field strength  $E$  and the electric potential  $V$  at points X and Y as shown in Figure 12A.

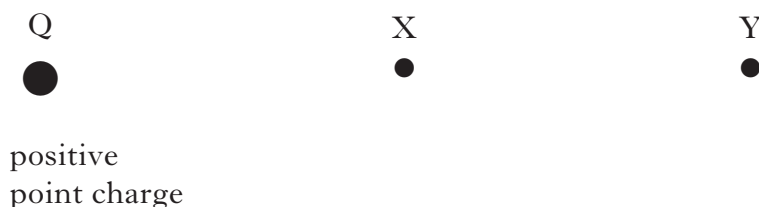


Figure 12A

The results of one set of measurements are shown in the table.

	X	Y
$E/\text{N C}^{-1}$	3.6	
$V/\text{V}$	7.2	3.6

- (a) The first student predicts that the electric field strength at Y will be  $1.8 \text{ N C}^{-1}$ , the second predicts it will be  $0.9 \text{ N C}^{-1}$ .

Which student has made the correct prediction?

You must justify your answer.

2

12. (continued)

- (b) In a second experiment a magnetised iron rod is placed into a coil of wire as shown in Figure 12B.

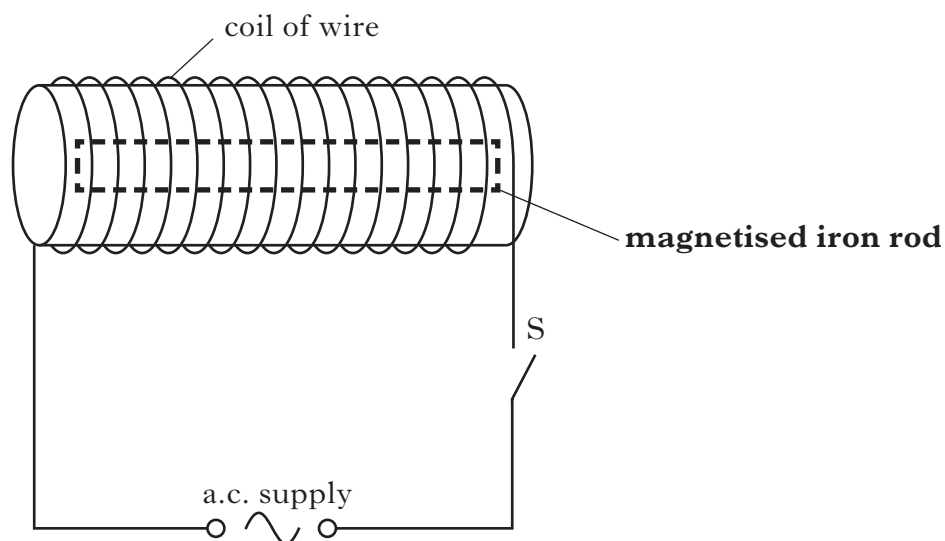


Figure 12B

Switch S is closed for a short time, passing an alternating current through the coil.

State the effect on the iron rod's magnetic field.

You must justify your answer.

3

(5)

[Turn over

13. An inductor of inductance  $4.0\text{ H}$  with negligible resistance is connected in series with a  $48\ \Omega$  resistor shown in Figure 13A.

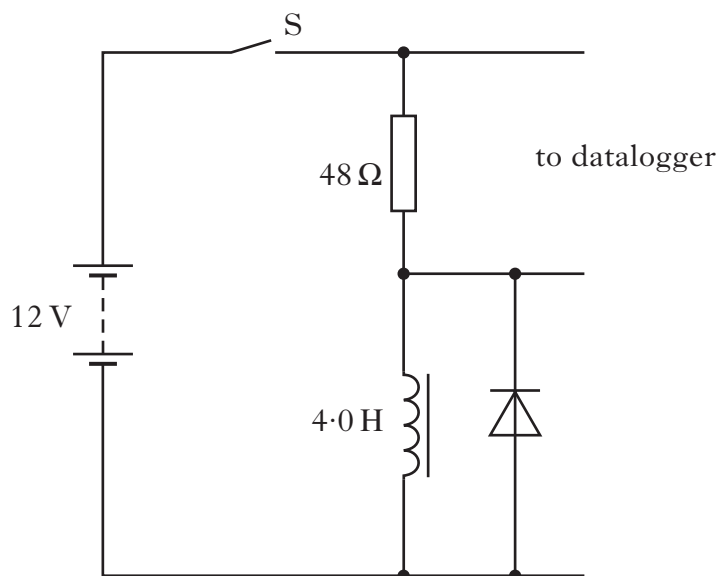


Figure 13A

The datalogger is set to display a graph of current against time.

- (a) Sketch the graph obtained from the time the switch S is closed until the current reaches a maximum. Numerical values are required on the current axis only. 2
- (b) Calculate the initial rate of change of current in the  $4.0\text{ H}$  inductor. 2
- (c) The  $4.0\text{ H}$  inductor is now connected in the circuit shown in Figure 13B.

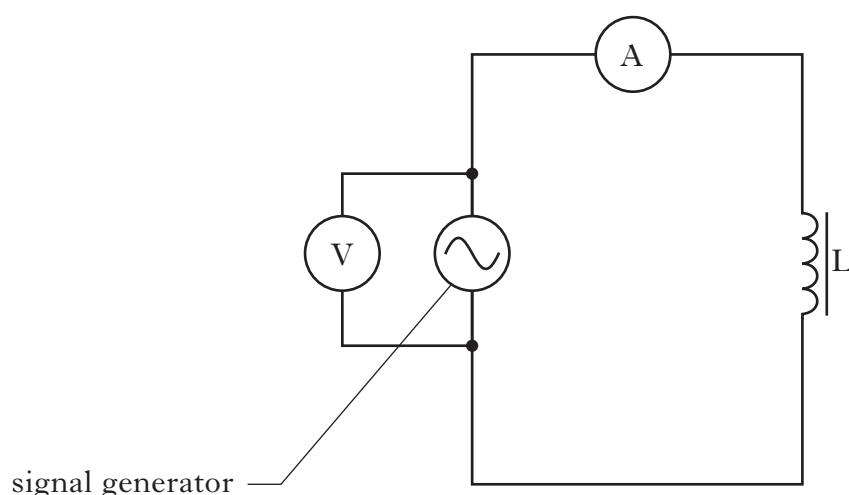


Figure 13B

The output voltage of the signal generator is set at  $6.0\text{ V}$ . The reading in the ammeter is  $5.0\text{ mA}$ .

Calculate the output frequency of the signal generator.

3  
(7)



14. A group of students were evaluating an experiment to investigate the relationship between the mass on a spring and its period of oscillation. Figure 14 shows some of the apparatus used.

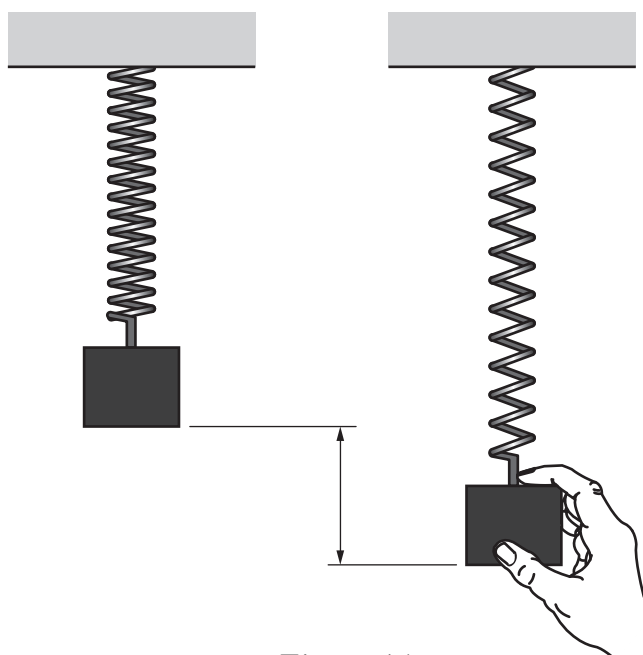


Figure 14

Student A stated “*I think we should use a balance that reads to 0.001 g instead of 0.1 g. This will give us a more accurate answer.*”

Student B stated “*I think we should repeat the time measurement and calculate a mean value.*”

Student C stated “*I think we should time the pendulum for 10 oscillations and divide this value by 10 to get the time for one complete oscillation. This will give us a more precise answer.*”

Student D stated “*I think it would be good to check the mass on another balance.*”

Using your knowledge of experimental analysis, comment on these statements. (3)

[Turn over

15. A student carries out an experiment to determine Young's modulus for a wire. The experimental set up is shown in Figure 15.

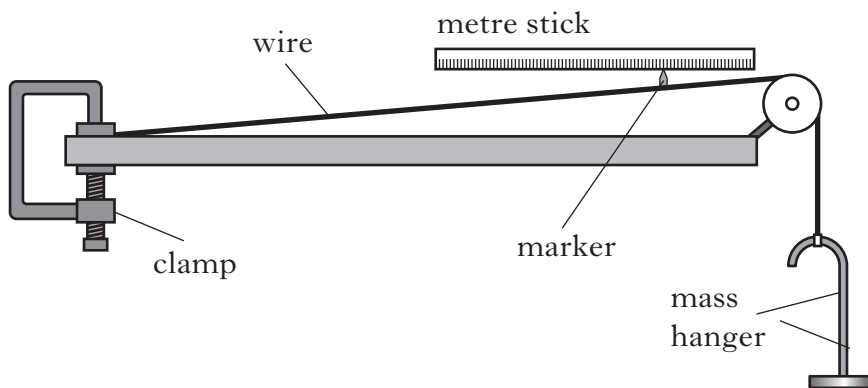


Figure 15

Young's modulus  $E$  can be determined by applying the formula

$$E = \frac{FL_o}{A_o\Delta L}.$$

Where  $F$  = force applied

$L_o$  = distance from clamp to marker

$\Delta L$  = length of extension of wire

$A_o$  = original cross sectional area of wire

The student attaches a mass hanger to the wire and fixes a marker on the wire at a distance of 2.00 m from the clamp. Masses are added to the hanger and the extension of the wire is measured by noting the distance moved by the marker along the metre stick. The masses are removed and the experiment repeated.

The length of the wire is measured using a tape measure.

The diameter of the wire is measured using a micrometer gauge.

## 15. (continued)

An extract from the student's workbook is shown.

Uncertainties : Combined scale and calibration

- tape measure  $\pm 5$  mm
- metre stick  $\pm 1$  mm
- balance  $\pm 0.1$  kg
- micrometer  $\pm 0.01$  mm

Mass (kg)	Force (N)	Wire extension (mm)			$E$ ( $\times 10^9 \text{ N m}^{-2}$ )
		1	2	Mean	
0.8	7.84	3	3	3	69
1.2	11.76	4	5	4.5	69
1.6	15.68	5	5	5	83
2.0	19.6	7	7	7	74
2.4	23.52	8	9	8.5	73

Wire diameter = 0.31 mm      Area =  $7.6 \times 10^{-8} \text{ N m}^{-2}$

Distance from clamp to marker = 2.00 m

Conclusion – The mean value of  $E$  is  $74 \times 10^9 \text{ N m}^{-2}$ .

(a) Suggest **two** improvements that could be made to the experimental procedures. 2

(b) Which quantity should the student measure more precisely to have the most significant impact on the percentage uncertainty?

Justify your answer. 2

(c) The student realises that calculating the mean of the individual measurements of  $E$  is an inappropriate method for estimating Young's modulus of the wire and so plots a graph of  $F$  against  $\Delta L$ .

Explain how the value of  $E$  can be determined from this graph. 2

**(6)**

[END OF QUESTION PAPER]

## ACKNOWLEDGEMENTS

Question 4(a) – 125702072 Shutterstock.com

Question 7(a) – 139591142 Shutterstock.com

# **X272/13/01**

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NATIONAL  
QUALIFICATIONS  
2014

THURSDAY, 22 MAY  
1.00 PM – 3.30 PM

PHYSICS  
ADVANCED HIGHER  
(Revised)

Relationships required for Advanced Higher Physics (Revised)

(For reference, relationships required for Higher Physics (Revised) are also included on *Page four*)

**Relationships required for Physics Advanced Higher (Revised)**

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a_t = r\alpha$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$T = Fr$$

$$T = I\alpha$$

$$L = mvr = mr^2\omega$$

$$L = I\omega$$

$$E_k = \frac{1}{2}I\omega^2$$

$$F = G \frac{Mm}{r^2}$$

$$V = -\frac{GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

apparent brightness,  $b = \frac{L}{4\pi r^2}$

Power per unit area =  $\sigma T^4$

$$L = 4\pi r^2 \sigma T^4$$

$$r_{\text{Schwarzschild}} = \frac{2GM}{c^2}$$

$$E = hf$$

$$\lambda = \frac{h}{p}$$

$$mvr = \frac{nh}{2\pi}$$

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

$$F = qvB$$

$$\omega = 2\pi f$$

$$a = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$y = A \cos \omega t \quad \text{or} \quad y = A \sin \omega t$$

$$v = \pm \omega \sqrt{(A^2 - y^2)}$$

$$E_K = \frac{1}{2} m \omega^2 (A^2 - y^2)$$

$$E_P = \frac{1}{2} m \omega^2 y^2$$

$$y = A \sin 2\pi \left( ft - \frac{x}{\lambda} \right)$$

$$\phi = \frac{2\pi x}{\lambda}$$

$$\text{optical path difference} = m\lambda \quad \text{or} \quad \left( m + \frac{1}{2} \right) \lambda$$

where  $m = 0, 1, 2, \dots$

$$\Delta x = \frac{\lambda l}{2d}$$

$$d = \frac{\lambda}{4n}$$

$$\Delta x = \frac{\lambda D}{d}$$

$$n = \tan i_p$$

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 r^2}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2}$$

$$V = \frac{Q}{4\pi \epsilon_0 r}$$

$$F = QE$$

$$V = Ed$$

$$F = IlB \sin \theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$t = RC$$

$$X_C = \frac{V}{I}$$

$$X_C = \frac{1}{2\pi f C}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$E = \frac{1}{2} LI^2$$

$$X_L = \frac{V}{I}$$

$$X_L = 2\pi f L$$

$$\frac{\Delta W}{W} = \sqrt{\left( \frac{\Delta X}{X} \right)^2 + \left( \frac{\Delta Y}{Y} \right)^2 + \left( \frac{\Delta Z}{Z} \right)^2}$$

$$\Delta W = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

## Relationships required for Physics Higher (Revised)

$$d = \bar{v}t$$

$$E_W = QV$$

$$V_{peak} = \sqrt{2}V_{rms}$$

$$s = \bar{v}t$$

$$E = mc^2$$

$$I_{peak} = \sqrt{2}I_{rms}$$

$$v = u + at$$

$$E = hf$$

$$Q = It$$

$$s = ut + \frac{1}{2}at^2$$

$$E_K = hf - hf_0$$

$$V = IR$$

$$v^2 = u^2 + 2as$$

$$E_2 - E_1 = hf$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$s = \frac{1}{2}(u + v)t$$

$$T = \frac{1}{f}$$

$$R_T = R_1 + R_2 + \dots$$

$$W = mg$$

$$v = f\lambda$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

$$F = ma$$

$$d\sin\theta = m\lambda$$

$$E = V + Ir$$

$$E_W = Fd$$

$$n = \frac{\sin\theta_1}{\sin\theta_2}$$

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V_S$$

$$E_P = mgh$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$E_K = \frac{1}{2}mv^2$$

$$\sin\theta_c = \frac{1}{n}$$

$$C = \frac{Q}{V}$$

$$P = \frac{E}{t}$$

$$I = \frac{k}{d^2}$$

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

$$p = mv$$

$$I = \frac{P}{A}$$

$$Ft = mv - mu$$

$$\text{path difference} = m\lambda \quad \text{or} \quad \left(m + \frac{1}{2}\right)\lambda \quad \text{where } m = 0, 1, 2, \dots$$

$$F = G\frac{Mm}{r^2}$$

$$\text{random uncertainty} = \frac{\text{max. value} - \text{min. value}}{\text{number of values}}$$

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$f_o = f_s \left(\frac{v}{v \pm v_s}\right)$$

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$z = \frac{v}{c}$$

$$v = H_0d$$