Wallace Hall Academy
Physics Department

Advanced Higher Physics

Astrophysics

Problems

Data

Common Physical Quantities

QUANTITY	SYMBOL	VALUE
Gravitational acceleration	g	9.8 m s^{-2}
Radius of Earth	R_E	$6.4 \times 10^6 \text{ m}$
Mass of Earth	M_E	$6.0 \times 10^{24} \text{ kg}$
Mass of Moon	M_M	$7.3 \times 10^{22} \text{ kg}$
Mean radius of Moon orbit		$3.84 \times 10^8 \text{ m}$
Universal constant of gravitation	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Speed of light in vacuum	c	$3.0 \times 10^8 \text{ m s}^{-1}$
Speed of sound in air	v	$3.4 \times 10^2 \text{ m s}^{-1}$
Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Charge on electron	e	$-1.60 \times 10^{-19} \text{ C}$
Mass of neutron	m_n	$1.675 \times 10^{-27} \text{ kg}$
Mass of proton	m_p	$1.673 \times 10^{-27} \text{ kg}$
Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$

Astronomical Data

Planet or satellite	Mass/ kg	Density/ kg m^{-3}	Radius/ m	Grav. accel./ m s^{-2}	Escape velocity/ m s^{-1}	Mean dist from Sun/ m	Mean dist from Earth/ m
Sun	1.99×10^{30}	1.41×10^3	7.0×10^8	274	6.2×10^5	--	1.5×10^{11}
Earth	6.0×10^{24}	5.5×10^3	6.4×10^6	9.8	11.3×10^3	1.5×10^{11}	--
Moon	7.3×10^{22}	3.3×10^3	1.7×10^6	1.6	2.4×10^3	--	3.84×10^8
Mars	6.4×10^{23}	3.9×10^3	3.4×10^6	3.7	5.0×10^3	2.3×10^{11}	--
Venus	4.9×10^{24}	5.3×10^3	6.05×10^6	8.9	10.4×10^3	1.1×10^{11}	--

Tutorial 1.0

Gravitation

1. State the inverse square law of gravitation.

2. Calculate the gravitational force between two cars parked 0.50 m apart. The mass of each car is 1000 kg.

3. Calculate the gravitational force between the Earth and the Sun.

4. (a) By considering the force on a mass, at the surface of the Earth, state the expression for the gravitational field strength, g, in terms of the mass and radius of the Earth.
   (b) (i) The gravitational field strength is 9.8 N kg^{-1} at the surface of the Earth. *Calculate* a value for the mass of the Earth.
   (ii) Calculate the gravitational field strength at the top of Ben Nevis, 1344 m above the surface of the Earth.
   (iii) Calculate the gravitational field strength at 200 km above the surface of the Earth.

5. (a) What is meant by the gravitational potential at a point?
   (b) State the expression for the gravitational potential at a point.
   (c) Calculate the gravitational potential:
   (i) at the surface of the Earth
   (ii) 800 km above the surface of the Earth.

6. 'A gravitational field is a conservative field.' Explain what is meant by this statement.

7. Calculate the energy required to place a satellite of mass 200 000 kg into an orbit at a height of 350 km above the surface of the Earth.

8. Which of the following are vector quantities:
   gravitational field strength, gravitational potential, escape velocity, universal constant of gravitation, gravitational potential energy, period of an orbit?

9. A mass of 8.0 kg is moved from a point in a gravitational field where the potential is –15 J kg^{-1} to a point where the potential is –10 J kg^{-1}.
   (a) What is the potential difference between the two points?
   (b) Calculate the change in potential energy of the mass.
   (c) How much work would have to be done against gravity to move the mass between these two points?

10. (a) Explain what is meant by the term 'escape velocity'.
    (b) Derive an expression for the escape velocity in terms of the mass and radius of a planet.
    (c) (i) Calculate the escape velocity from both the Earth and from the Moon.
    (ii) Using your answers to (i) comment on the atmosphere of the Earth and the Moon.

11. Calculate the gravitational potential energy and the kinetic energy of a 2000 kg satellite in geostationary orbit above the Earth.

12. (a) A central force required to keep a satellite in orbit. Derive the expression for the orbital period in terms of the orbital radius.
    (b) A satellite is placed in a parking orbit above the equator of the Earth.
    (i) State the period of the orbit.
    (ii) Calculate the height of the satellite above the equator.
    (iii) Determine the linear speed of the satellite.
    (iv) Find the central acceleration of the satellite.

13. A white dwarf star has a radius of 8000 km and a mass of 1.2 solar masses.
    (a) Calculate the density of the star in kg m^{-3}.
    (b) Find the gravitational potential at a point on the surface.
    (c) Calculate the acceleration due to gravity at a point on the surface.
    (d) *Estimate* the potential energy required to raise your centre of gravity from a sitting position to a standing position on this star.
    (e) A 2 kg mass is dropped from a height of 100 m on this star.
    How long does it take to reach the surface of the star?

14. (a) How are photons affected by a massive object such as the Sun?
    (b) Explain, using a sketch, why light from a distant star passing close to the Sun may suggest that the star is at a different position from its ‘true’ position.
    (c) Explain what is meant by the term ‘black hole’.

Tutorial 1.1

Gravitation

1 Show that the force of attraction between two large ships of mass 50000 tonnes and separated by a distance of 20 m is 417 N. (1 tonne = 1000 kg)

2 Calculate the gravitational force of attraction between the proton and the electron in a hydrogen atom. Assume the electron is describing a circular orbit with a radius of 5.3×10^{-11} m.
(mass of proton = 1.67×10^{-27} kg; mass of electron = 9.11×10^{-31} kg).

3 A satellite, of mass 1500 kg, is moving at constant speed in a circular orbit 160 km above the Earth's surface.
(a) Calculate the period of rotation of the satellite.
(b) Calculate the total energy of the satellite in this orbit.
(c) Calculate the minimum amount of extra energy required to boost this satellite into a geostationary orbit which is at a distance of 36 000 km above the Earth's surface.

4 The planet Mars has a mean radius of 3.4×10^{6} m. The Earth's mean radius is 6.4×10^{6} m. The mass of Mars is 0.11 times the mass of the Earth.
(a) How does the mean density of Mars compare with that of the Earth?
(b) Calculate the value of "g" on the surface of Mars.
(c) Calculate the escape velocity on Mars.

5 Determine the potential energy between the planet Saturn and its rings.
The mass of Saturn is 5.72×10^{26} kg. The rings have a mass of 3.5×10^{18} kg and are concentrated at an average distance of 1.1×10^{8} m from the centre of Saturn.

6 During trial firing of Pioneer Moon rockets, one rocket reached an altitude of 125 000 km. Neglecting the effect of the Moon, estimate the velocity with which this rocket struck the atmosphere of the Earth on its return. (Assume that the rocket's path is entirely radial and that the atmosphere extends to a height of 130 km above the Earth's surface).

7 (a) Sketch the gravitational field pattern between the Earth and Moon.
(b) Gravity only exerts attractive forces. There should therefore be a position between the Earth and Moon where there is no gravitational field - a so-called 'null' point.
By considering the forces acting on a mass m placed at this point, calculate how far this position is from the centre of the Earth.

8 Mars has two satellites named Phobos and Deimos. Phobos has an orbital radius of 9.4×10^{6} m and an orbital period of 2.8×10^{4} s.
Using Kepler's third law ($\frac{r^3}{T^2}$ = constant), calculate the orbital period of Deimos which has an orbital radius of 2.4×10^{7} m.

9 When the Apollo 11 satellite took the first men to the Moon in 1969 its trajectory was very closely monitored.
The satellite had a velocity of 5374 m s^{-1} when 26306 km from the centre of the Earth and this had dropped to 3560 m s^{-1} when it was 54368 km from the centre of the Earth. The rocket motors had **not** been used during this period.
Calculate the gravitational potential difference between the two points. Remember that the unit of gravitational potential is J kg^{-1}.

10 Show that an alternative expression for the escape velocity from a planet may be given by:

$\boxed{v_e = \sqrt{2gR}}$ where g = the planet's surface gravitational attraction and R = the radius of the planet.

11 The Escape Velocity for the Earth $v_e = \sqrt{\dfrac{2GM_E}{r_E}}$ or $v_e = \sqrt{2gr_E}$

Using data on the Earth, show that the escape velocity equals 1.1×10^4 m s^{-1}, or 11 km s^{-1}.

12 Show that a satellite orbiting the Earth at a height of 400 km has an orbital period of 93 minutes. Note that a height of 400 km is equal to a radius of R_E + 400 km.

13 (a) A geostationary orbit has a period of approximately 24 hours. Find the orbital radius for a satellite in such an orbit.
(b) Hence find the height of this satellite above the Earth.
(c) Show on a sketch of the Earth the minimum number of geostationary satellites needed for world-wide communication.

Tutorial 2.0

Space and time

1. What is meant by an inertial frame of reference?

2. What is meant by a non-inertial frame of reference?

3. Which frame of reference applies to the theory of special relativity (studied in Higher Physics)?

4. At what range of speeds do the results obtained by the theory of special relativity agree with those of Newtonian mechanics?

5. How does the equivalence principle link the effects of gravity with acceleration?

6. In which part of an accelerating spacecraft does time pass more slowly?

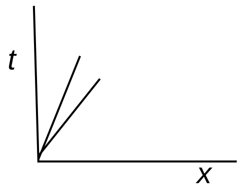
7. Does time pass more quickly or more slowly at high altitude in a gravitational field?

8. How many dimensions are normally associated with space-time?

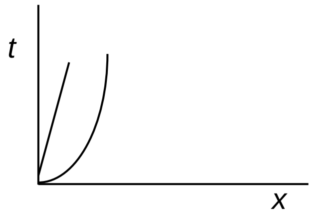
9. Two space-time diagrams are shown, with a worldline on each. Write down what each of the worldlines describes.

(a) (b)

10. The space-time diagram shows two worldlines. Which worldline describes a faster speed? (These speeds are much less than the speed of light.)

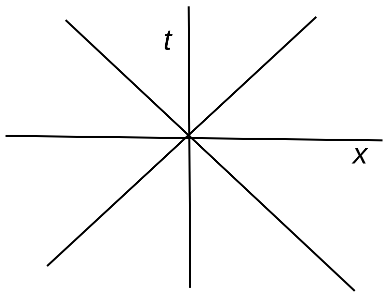


11. Explain the difference between these two worldlines on the space-time diagram.



12. Explain what is meant by the term *geodesic*.

13. Copy the following space-time diagram.



Insert the following labels onto your diagram:

- the present
- the future
- the past
- $v = c$
- $v < c$
- $v > c$.

14. What effect does mass have on spacetime?

15. Describe two situations where a human body experiences the sensation of force.

16. How does general relativity interpret the cause of gravity?

17. Mercury's orbit around the Sun could not be predicted accurately using classical mechanics. General relativity was able to predict Mercury's orbit accurately. Investigate this using a suitable search engine and write a short paragraph summarising your results.

18. A star of mass 4.5×10^{31} kg collapses to form a black hole. Calculate the Schwarzschild radius of this black hole.

19. A star of mass equivalent to six solar masses collapses to form a black hole. Calculate the Schwarzschild radius of this black hole.

20. If our Sun collapsed to form a black hole, what would be the Schwarzschild radius of this black hole?

21. If our Earth collapsed to form a black hole, what would be the Schwarzschild radius of this black hole?

22. A star is approximately the same size as our Sun and has an average density of 2.2×10^3 kg m^{-3}. If this star collapsed to form a black hole, calculate the Schwarzschild radius of the black hole.

Tutorial 3.0

Stellar physics

1. A star emits electromagnetic radiation with a peak wavelength of 6.8×10^{-7} m.

   (a) Use Wien's law ($\lambda_{max}T = 3 \times 10^{-3}$) to calculate the surface temperature of the star.
   (b) Calculate the power of the radiation emitted by each square metre of the star's surface where the star is assumed to be a black body.
   Stefan–Boltzmann constant = 5.67×10^{-8} J s^{-1} m^{-2} K^{-4}.

2. The Sun has a radius of 7.0×10^{8} m and a surface temperature of 5800 K.

   (a) Calculate the power emitted per m^2 from the Sun's surface.
   (b) Calculate the luminosity of the Sun.
   (c) Calculate the apparent brightness of the Sun as seen from the Earth.

3. Three measurements of a distant star are possible from Earth.
   These measurements are:

apparent brightness	= 4.3×10^{-9} W m^{-2}
peak emitted wavelength	= 2.4×10^{-7} m
distance to star (parallax method)	= 8.5×10^{17} m

   (a) Use Wien's law ($\lambda_{max}T = 3 \times 10^{-3}$) to calculate the surface temperature of the star.
   (b) Calculate the energy emitted by each square metre of the star's surface per second.
   (c) Calculate the luminosity of the star.
   (d) Calculate the radius of the star.

4. A star is 86 ly from Earth and has a luminosity of 4.8×10^{28} W m^{-2}.
   Calculate the apparent brightness of the star.

5. The apparent brightness of a star is 6.2×10^{-8} W m^{-2}. The star is 16 ly from Earth.
   Calculate the luminosity of the star.

6. A star with luminosity 2.1×10^{30} W m^{-2} has an apparent brightness of 7.9×10^{-8} W m^{-2} when viewed from Earth.
   Calculate the distance of the star from Earth:
   (a) in metres
   (b) in light years.

7. A star with radius 7.8×10^{8} m and surface temperature 6300 K has an apparent brightness of 1.8×10^{-8} W m^{-2}.
   Calculate its distance from the Earth.

8. A star with radius 9.5×10^{9} m and surface temperature 5900 K is 36 ly from Earth.
   Calculate the apparent brightness of the star.

9. Show mathematically that the luminosity of a star varies directly with the square of its radius and the fourth power of its surface temperature.

10. Show mathematically that the apparent brightness of a star varies directly with the square of its radius and the fourth power of its surface temperature and varies inversely with the square of its distance from the Earth.

11. Two stars, A and B, are the same distance from the Earth.
The apparent brightness of star A is 8.0×10^{-12} W m^{-2} and the apparent brightness of star B is 4.0×10^{-13} W m^{-2}.
Show that star A has 20 times the luminosity of star B.

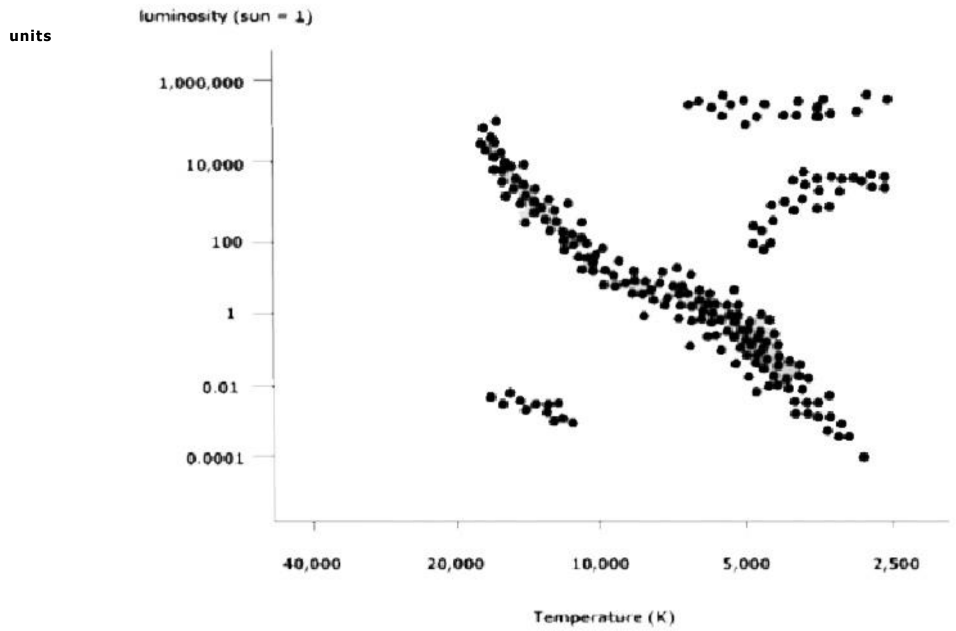
12. A star has half of our Sun's surface temperature and 400 times our Sun's luminosity.
How many times bigger is the radius of this star compared to the Sun?

13. Information about two stars A and B is given below.

surface temperature of star	A = 3 × surface temperature of star B
radius of star	A = 2 × radius of star B

(a) How many times is the luminosity of star A greater than the luminosity of star B?
(b) Stars A and B have the same apparent brightness from Earth.
Which star is furthest from Earth and by how many times?

14. The diagram shows one way of classifying stars. Each dot on the diagram represents a star.



(a) What name is usually given to this type of diagram?
(b) The stars are arranged into four main regions. Identify the region called:
  (i) the main sequence
  (ii) giants
  (iii) super giants
  (iv) white dwarfs.

(c) (i) In which of the regions on the diagram is the Sun?

(ii) The surface temperature of the Sun is approximately 5800 K. Explain why the scale on the temperature axis makes it difficult to identify which dot represents the Sun.

(d) In which region would you find the following:

(i) a hot bright star

(ii) a hot dim star

(iii) a cool bright star

(iv) a cool dim star?

(e) A star is cooler than, but brighter than the Sun.

(i) What can be deduced about the size of this star compared to the size of the Sun?

(ii) What region would this star be in?

(f) A star is hotter than, but dimmer than, the Sun.

(i) What can be deduced about the size of this star compared to the size of the Sun?

(ii) What region would this star be in?

(g) The Sun's nuclear fuel will be used up with time. What will then happen to the Sun's position in the above diagram?