Wallace Hall Academy
Physics Department

Advanced Higher Physics

Astrophysics

Solutions

Tutorial 1.0

Gravitation

1. $F = \frac{Gm_1m_2}{r^2}$

2. $F = \frac{6.67 \times 10^{-11} \times 1000 \times 1000}{0.5^2}$ = 2.7×10^{-4} N

3. $F = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1.99 \times 10^{30}}{(1.5 \times 10^{11})^2}$ = 3.5×10^{22} N

4. (a) $F = m_1 g = \frac{Gm_1m_2}{r^2}$ giving $g = \frac{G\,M}{R^2}$ where M is the mass of the Earth

   (b) (i) $9.8 = \frac{6.67 \times 10^{-11} \times M}{(6.4 \times 10^6)^2}$ giving M = 6.0×10^{24} kg

   (ii) $g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6 + 1344)^2}$ = 9.8 N kg^{-1} (No noticeable difference!)

   (iii) $g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6 + 200 \times 10^3)^2}$ = 9.2 N kg^{-1}

5. (a) The work done to move 1 kg from infinity to that point.

   (b) $V_p = -\frac{G\,m}{r}$

   (c) (i) $V_p = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}$ = -6.2×10^7 J kg^{-1}

   (ii) $V_p = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6 + 800 \times 10^3}$ = -5.6×10^7 J kg^{-1}

   (The negative sign indicates that the potential at infinity is higher than the potential on or close to the Earth.)

6. The work done *by the force of gravity* on a particle which moves through a round trip back to its starting point is zero. Energy is conserved.

7. Energy required = $E_k + E_p$ and $E_k = ½\,mv^2$ but $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$$= \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right) = -\frac{GMm}{2r} = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^5}{2(6.4 \times 10^6 + 350 \times 10^3)}$$

= -5.9×10^{12} J (The negative sign indicates a decrease in energy.)

8. Gravitational field strength, escape velocity

9. (a) -10 – (-15) = 5 J kg^{-1}
   (b) 5 x 8 = 40 J
   (c) 40 J

10 (a) The minimum velocity a mass must have that would allow it to escape from the gravitational field of the planet or star.

(b) Gravitational E_p at surface = $-\frac{GMm}{r}$ E_p(Infinity) = 0

Kinetic energy at surface = ½ mv^2 E_k(Infinity) = 0

Total energy at surface = total energy at infinity

$-\frac{GMm}{r} + \frac{1}{2}mv^2 = 0$ thus $v^2 = \frac{2GM}{r}$ and $v_{escape} = \sqrt{\frac{2GM}{r}}$

(c) (i) v_e(Earth) = 1.1×10^4 m s^{-1} v_e(Moon) = 2.4×10^3 m s^{-1}

(ii) Escape velocity on the Moon is very small so there will be no atmosphere.

11. For a geostationary satellite T = 24 x 60 x 06 s and $T = 2\pi\sqrt{\frac{r^3}{GM}}$

for the Earth this gives r = 4.2×10^7 m, where r is from the centre of the Earth

$E_p = -\frac{GMm}{r} = -1.9 \times 10^{11}$ J $E_k = \frac{GMm}{2r} = 0.95 \times 10^{11}$ J

12. (a) See Mechanics – Student Material page 25

(b) (i) 24 hours (24 x 60 x 60 s)

(ii) radius of orbit = 4.2×10^7 m, (using the equation for T as in Q 11.)

height above equator = $4.2 \times 10^7 - 6.0 \times 10^6 = 3.6 \times 10^7$ m

(iii) $v = r\omega = r\frac{2\pi}{T} = 3.05 \times 10^3$ m s^{-1}

(iv) central acceleration = $\frac{v^2}{r}$ = 0.22 rad s^{-2}

13. (a) density = $\frac{1.2 \times 1.99 \times 10^{30}}{(4/3)\pi(8000 \times 10^3)^2}$ = 1.1×10^9 kg m^{-3}

(b) Vp = $-\frac{G\,m}{r}$ = -1.99×10^{13} J kg^{-1}

(c) g = $\frac{G\,m}{r^2}$ = 2.5×10^6 N kg^{-1}

(d) Assume a change of height of 1 m and a mass of 60 kg

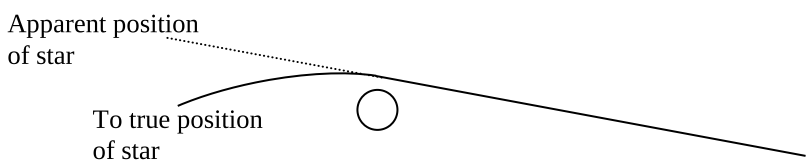
E_p = mgh = 60 x 2.5×10^6 x 1 = 1.5×10^8 J

(e) s = ut + ½ at^2 giving 100 = 0 + ½ x 2.5×10^6 x t^2

t = 8.9×10^{-3} s (Notice that this does not depend on the mass)

14. (a) There is a gravitational force of attraction on the photons. The photon path would be deflected.

(b)



The light from the star is deflected by the Sun. The observer might consider that the star position is in a straight line from his line of sight.

(c) If a dense star has an escape velocity greater than the speed of light, then no light could escape and the star would appear ‘black’.

Tutorial 1.1

Gravitation

1 Use $F = \dfrac{G m_1 m_2}{r^2} = \dfrac{6.67 \times 10^{-11} \times (50000 \times 10^3)^2}{20^2}$

$$F = \underline{\underline{417}}\ \text{N}$$

2 $F = \dfrac{G m_p m_e}{r^2} = \dfrac{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 9.11 \times 10^{-31}}{(5.3 \times 10^{-11})^2}$

$$= \underline{\underline{3.6 \times 10}}^{-47}\ \text{N}$$

3 (a) The central force is supplied by the force of gravity

$$\frac{m v^2}{r} = \frac{G M m}{r^2}$$

$$v^2 = \frac{G M}{r} \qquad \text{but } v = \frac{2 \pi r}{T}$$

$$T^2 = \frac{4 \pi^2 r^3}{G M} \qquad \text{(Notice } r = R_E + 160 \text{ km)}$$

$$T = \sqrt{\frac{4 \pi^2 r^3}{G M}} = \sqrt{\frac{4\pi^2 \times (6.4 \times 10^6 + 160 \times 10^3)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}}$$

$$= 5277 \text{ seconds}$$

$$= \underline{\underline{88}} \text{ minutes}$$

(b) $E_{tot} = E_p + E_k = -\dfrac{G M m}{2 r} = -\dfrac{}{2 \times (6.4 \times 10^6 + 160 \times 10^3)}$

$$= \underline{\underline{-4.58 \times 10}}^{10}\ \text{J}$$

(c) E_{tot} at 36000 km $= -\dfrac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1500}{2 \times (6.4 \times 10^6 + 36000 \times 10^3)}$

$$= -0.71 \times 10^{10}\ \text{J}$$

Minimum energy required is the difference between these two.

Energy required $= -0.71 \times 10^{10} - (-4.58 \times 10^{10})$

$$= \underline{\underline{3.87 \times 10}}^{10}\ \text{J}$$

4 (a) Density, $\rho = \frac{M}{V}$; volume of sphere $= \frac{4}{3}\pi r^3$

$$\rho_{mars} = \frac{M_m}{V_m} = \frac{0.11\,M_e}{\frac{4}{3}\pi r_e^3} \qquad \rho_{earth} = \frac{M_e}{V_e} = \frac{M_e}{\frac{4}{3}\pi r_e^3}$$

$$\frac{\rho_{mars}}{\rho_{earth}} = \frac{M_m \times r_e^3}{r_m^3 \times M_e} = \frac{0.11 \times M_e \times r_e^3}{r_m^3 \times M_e} = 0.11 \times \left[\frac{r_e}{r_m}\right]^3$$

$$= 0.11 \times (6.4 \times 10^6 / 3.4 \times 10^6)^3$$

$$\rho_{mars} = \underline{0.73}\ \rho_{earth}$$

(b) At the surface of any planet: $mg = \frac{G\,M\,m}{r^2}$ m cancels

$$g_{mars} = \frac{G\,M_{mars}}{r^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 0.11}{(3.4 \times 10^6)^2}$$

$$= \underline{3.8}\ \text{m s}^{-2}$$

(c) $$v_{esc} = \sqrt{\frac{2\,G\,M}{r}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 0.11}{3.4 \times 10^6}}$$

$= 5.1 \times 10^3$ m s^{-1}

escape velocity on Mars is $\underline{5.1}$ km s^{-1}

5 $$E_p = -\frac{G\,M\,m}{r}$$ M = mass of Saturn; m = mass of rings

$$= -\frac{6.67 \times 10^{-11} \times 5.72 \times 10^{26} \times 3.5 \times 10^{18}}{1.1 \times 10^8}$$

$$= -\ \underline{1.21 \times 10^{27}}\ \text{J}$$

6 E_p at highest point $= -\frac{G\,M\,m}{r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times m}{(6.4 \times 10^6 + 125 \times 10^6)}$

$= -\ (3.05 \times 10^6 \times m)$ J

E_p at height of atmosphere $= -\frac{G\,M\,m}{r} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times m}{(6.4 \times 10^6 + 130 \times 10^3)}$

$= -\ (61.29 \times 10^6 \times m)$ J

Energy difference -> E_k thus $\Delta E_p = \frac{1}{2} m v^2$

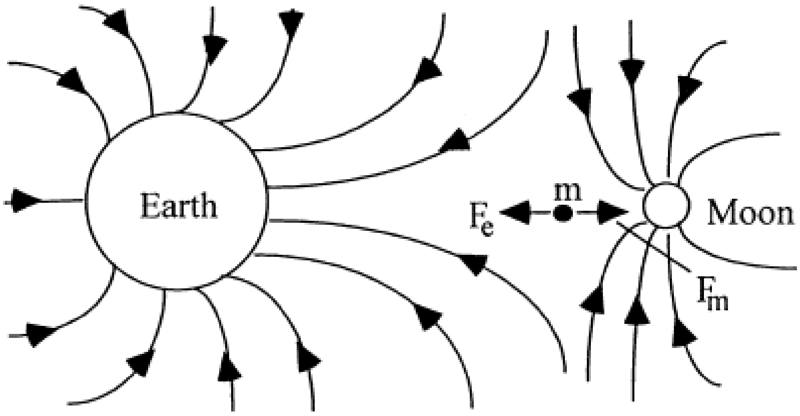
$-\ (3.05 \times 10^6 \times m) - (-\,61.29 \times 10^6 \times m) = \frac{1}{2} m v^2$

$58.24 \times 10^6 \times m = \frac{1}{2} m v^2$ m cancels

$v^2 = 2 \times 58.24 \times 10^6$

$v = \underline{10.8}$ km s^{-1}

7 (a)



(b) Let r_1 = distance of m from centre of the Earth and
let r_2 = distance of m from centre of the Moon.

Thus $r_1 + r_2$ = separation of the Earth and Moon = 3.84×10^8 m

If m is at the 'null' point: $F_e = F_m$

$$\frac{G M_e m}{r_1^2} = \frac{G M_m m}{r_2^2} \qquad \text{(G and m cancel)}$$

$$\frac{r_1^2}{r_2^2} = \frac{M_e}{M_m} \quad \text{(take square roots of both sides)}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{M_e}{M_m}} = \sqrt{\frac{6 \times 10^{24}}{7.3 \times 10^{22}}} = 9.07$$

$r_1 = 9.07 \times r_2$ and $r_2 = 3.84 \times 10^8 - r_1$

$r_1 = 9.07 \times (3.84 \times 10^8 - r_1) = 9.07 \times 3.84 \times 10^8 - 9.07\, r_1$

$$r_1 = \frac{9.07 \times 3.84 \times 10^8}{(9.07 + 1)}$$

$= \underline{3.5 \times 10^8}$ m

The null point in this field, (ignoring the effect of the Sun), is approximately $\frac{9}{10}$ of the distance from the centre of the Earth to the moon.

8 Use Kepler's Third Law: $\frac{r^3}{T^2}$ = constant for a gravitational system

Thus, for Mars: $\frac{r_P^3}{T_P^2} = \frac{r_D^3}{T_D^2}$ P = Phobos, D = Deimos

$$T_D = \sqrt{\frac{(2.4 \times 10^7)^3 \times (2.8 \times 10^4)^2}{(9.4 \times 10^6)^3}}$$

$= \underline{1.14 \times 10^5}$ s (= 31.7 hours)

9 Change in E_k = ΔE_k = $\frac{1}{2}$ m x $(5374)^2$ - $\frac{1}{2}$ m x $(3560)^2$

ΔE_k per unit mass, $\frac{\Delta E_k}{m}$ = $(14.44 - 6.34) \times 10^6$ J kg^{-1}

= 8.10×10^6 J kg^{-1}

This must be equivalent to the change in gravitational potential.

ΔV = $\underline{8.10 \times 10}^6$ J kg^{-1}

10 For escape velocity: $\frac{1}{2} m v^2 = \frac{G M m}{R}$

also $mg = \frac{G M m}{R^2}$ m cancels in both equations

$\frac{1}{2} v^2 = \frac{G M}{r}$ ---------- 1 & $g R = \frac{G M}{r}$ ------2

combine 1 and 2: $\frac{1}{2} v^2 = g R$ giving $v^2 = 2 g R$

$v = \sqrt{2 g R}$ as required.

11 $v_{esc} = \sqrt{\frac{2 G M_E}{r_E}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}}$

v_{esc} = $\underline{1.1 \times 10}^4$ m s^{-1}

or $v_{esc} = \sqrt{2 g r_E} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$ = 11 km s^{-1}

12 $T = 2\pi\sqrt{\frac{r^3}{G M}}$ (see question 3 (a) for obtaining equation)

$= 2\pi\sqrt{\frac{6.4 \times 10^6 + 400 \times 10^3}{6.67 \times 10^{11} \times 6 \times 10^{24}}}$

= = 5569 s = $\underline{93}$ minutes

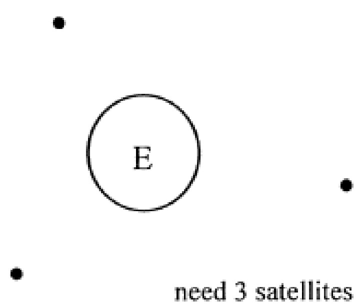
13 (a) $r^3 = \frac{T^2}{4\pi^2} . G M$ (see question 3 (a) for obtaining equation)

$= \frac{(24 \times 60 \times 60)^2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4\pi^2}$

r = $\underline{42 \times 10}^6$ m

(b) Thus height = $42.3 \times 10^6 - 6.4 \times 10^6 = 35.9 \times 10^6$ height = $\underline{36 \times 10}^6$ m

(c)



need 3 satellites at least

Tutorial 2.0

Space and time

Numerical answers:

18. 6.67×10^4 m

19. 1.8×10^4 m

20. 3000 m

21. 8.9×10^{-3} m (8.9 mm)

22. 4.7×10^3 m

Tutorial 3.0

Stellar physics

Numerical answers:

1. (a) 4410 K
   (b) 2.1×10^{7} W m^{-2}

2. (a) 6.42×10^{7} W m^{-2}
   (b) 3.95×10^{26} W
   (c) 1.4×10^{3} W m^{2}

3. (a) 12500 K
   (b) 1.38×10^{9} J (m^{-2} s^{-1})
   (c) 3.9×10^{28} W
   (d) 1.5×10^{9} m

4. 5.8×10^{-9} W m^{-2}

5. 1.8×10^{28} W m^{-2}

6. (a) 1.45×10^{18} m
   (b) 153 ly

7. 5.5×10^{16} m (5.8 ly)

8. 5.3×10^{-8} W m^{-2}

9. Show $L = 4\pi r^2 \sigma T^4$

10. Show that apparent brightness = $\sigma r^2 T^4 / d^2$

12. 80

13. (a) 324
    (b) Star A; 18 times more distant than star B.