Wallace Hall Academy
Physics Department

Advanced Higher Physics

Rotational motion

Problems

Data

Common Physical Quantities

QUANTITY	SYMBOL	VALUE
Gravitational acceleration	g	9.8 m s^{-2}
Radius of Earth	R_E	6.4 x 10^6 m
Mass of Earth	M_E	6.0 x 10^{24} kg
Mass of Moon	M_M	7.3 x 10^{22} kg
Mean radius of Moon orbit		3.84 x 10^8 m
Universal constant of gravitation	G	6.67 x 10^{-11} m^3 kg^{-1} s^{-2}
Speed of light in vacuum	c	3.0 x 10^8 m s^{-1}
Speed of sound in air	v	3.4 x 10^2 m s^{-1}
Mass of electron	m_e	9.11 x 10^{-31} kg
Charge on electron	e	-1.60 x 10^{-19} C
Mass of neutron	m_n	1.675 x 10^{-27} kg
Mass of proton	m_p	1.673 x 10^{-27} kg
Planck's constant	h	6.63 x 10^{-34} J s
Permittivity of free space	ε_0	8.85 x 10^{-12} F m^{-1}
Permeability of free space	μ_0	4π x 10^{-7} H m^{-1}

Astronomical Data

Planet or satellite	Mass/ kg	Density/ kg m^{-3}	Radius/ m	Grav. accel./ m s^{-2}	Escape velocity/ m s^{-1}	Mean dist from Sun/ m	Mean dist from Earth/ m
Sun	1.99x 10^{30}	1.41 x 10^3	7.0 x 10^8	274	6.2 x 10^5	--	1.5 x 10^{11}
Earth	6.0 x 10^{24}	5.5 x 10^3	6.4 x 10^6	9.8	11.3 x 10^3	1.5 x 10^{11}	--
Moon	7.3 x 10^{22}	3.3 x 10^3	1.7 x 10^6	1.6	2.4 x 10^3	--	3.84 x 10^8
Mars	6.4 x 10^{23}	3.9 x 10^3	3.4 x 10^6	3.7	5.0 x 10^3	2.3 x 10^{11}	--
Venus	4.9 x 10^{24}	5.3 x 10^3	6.05 x 10^6	8.9	10.4 x 10^3	1.1 x 10^{11}	--

TUTORIAL 1.0

Equations of motion

1 The displacement, s in metres, of an object after a time, t in seconds, is given by

$$s = 90t - 4\,t^2$$

(a) Find by differentiation the equation for its velocity.
(b) At what time will the velocity be zero?
(c) Show that the acceleration is a constant and state its value.

2 Given that $a = \frac{dv}{dt}$, show by integration that the velocity, v, is given by

$$v = u + at.$$

State clearly the meaning of the symbol, u, in this equation.

3. Given that $v = \frac{ds}{dt}$ and $v = u + at$, show by integration that

$$s = ut + \tfrac{1}{2}at^2.$$

Where the symbols have their usual meaning.

4. The displacement, s, of a moving object after a time, t, is given by

$$s = 8 - 10t + t^2.$$

Show that the unbalanced force acting on the object is constant.

5. The displacement, s, of an object after time, t, is given by $s = 3t^3 + 5t$.
(a) Derive an expression for the acceleration of the object.
(b) Explain why this expression indicates that the acceleration is not constant.

6. A trolley is released from the top of a runway which is 6 m long.
The displacement, s in metres, of the trolley is given by the expression

$$s = 5t + t^2, \text{ where t is in seconds.}$$

Determine:
(a) an expression for the velocity of the trolley
(b) the acceleration of the trolley
(c) the time it takes the trolley to reach the bottom of the runway
(d) the velocity of the trolley at the bottom of the runway.

7. A box slides down a smooth slope with an acceleration of 4 m s^{-2}. The velocity of the box at a time t = 0 is 3 m s^{-1} down the slope.
Using $a = \frac{dv}{dt}$ show by integration that the velocity, v, of the box is given by

$$v = 3 + 4t.$$

8. The equation for the velocity, v, of a moving trolley is $v = 2 + 6t$.
Using $v = \frac{ds}{dt}$ derive an expression for the displacement, s, of the trolley.

9 A projectile is launched from the top of a building with an initial speed of 20 m s^{-1} at an angle of 30° to the horizontal. The height of the building is 30 m.

(a) Calculate how long it takes the projectile to reach the ground.
(b) Calculate the velocity of the projectile on impact with the ground, (magnitude and direction).

TUTORIAL 2.0

Angular motion

1. Convert the following from degrees to radians:
   30^o, 45^o, 60^o, 90^o, 180^o, 270^o, 360^o, 720^o.

2. Convert the following from radians to degrees:
   1 rad, 10 rad, 0.1 rad, π rad, 2π rad, ½ π rad, $\frac{\pi}{6}$ rad.

3. Convert the following from revolutions per minute to radians per second:
   33 rpm, 45 rpm, 78 rpm, 300 rpm.

4. Using calculus notation write down the expression for
   (a) the angular velocity in terms of the angular displacement
   (b) the angular acceleration in terms of the angular velocity
   (c) the angular acceleration in terms of the angular displacement.

5. State the three equations which can be used when an object moves with a constant angular acceleration, α.
   State the meaning of each symbol used.

6. A disc is slowed uniformly at 5.0 rad s^{-2} for 4.0 s. The initial angular velocity is 200 rad s^{-1}.
   (a) Determine the angular velocity at the end of the four seconds.
   (b) What is the angular displacement in this time?

7. The angular velocity of an engine is increased from 800 rpm to 3 000 rpm in 8.0 s.
   (a) Determine the angular acceleration. You may assume this is uniform.
   (b) Find the total angular displacement.
   (c) How many revolutions does the engine make during this 8.0 s?

8. A wheel accelerates uniformly from rest at 3.0 rad s^{-2} for 5.0 s.
   (a) Find
      (i) the final angular velocity after 5.0 s
      (ii) the angular displacement after 5.0 s.
   (b) The wheel has a radius of 1.50 m.
      Determine the linear velocity at a point on its rim at the end of the 5.0 s.

9. | | |
   |---|---|
   | Radius of Earth = 6.4×10^3 km | Geostationary orbit radius = 3.6×10^4 km |
   | Radius of Earth's orbit = 1.5×10^8 km | Radius of Moon's orbit = 3.8×10^5 km |
   | Period of Earth about Sun = 365 days | Period of Moon about Earth = 28 days |

   (a) Calculate the angular velocity in rad s^{-1} of
      (i) the Earth about the sun
      (ii) the Moon about the Earth
      (iii) an object on the Earth's surface about its axis of rotation
      (iv) a geostationary satellite.
   (b) Find the tangential velocity in m s^{-1} of each of the above quantities in part (a).

10. Derive the expression $v = r\omega$ for a particle in circular motion.

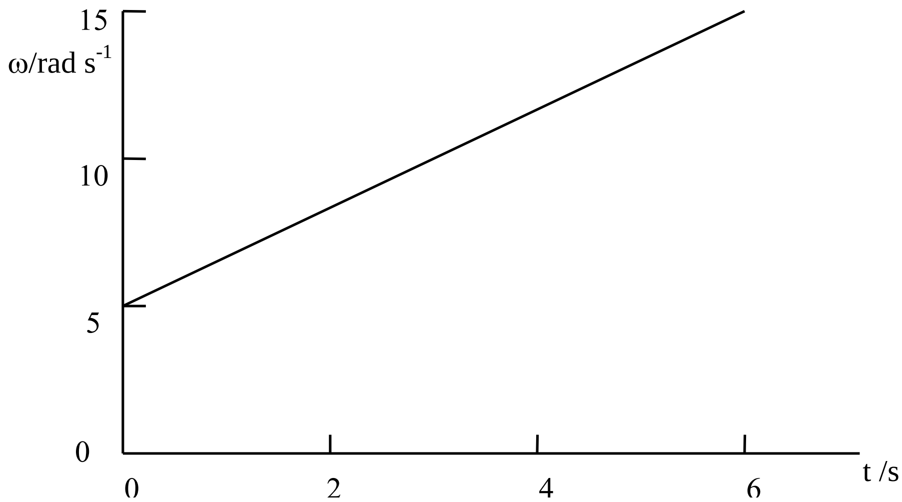
TUTORIAL 2.1

Angular Motion

1 If 2π radians equals 360°, calculate the number of degrees in one radian.

2 Calculate the angular velocity in rad s^{-1} of the second hand of an analogue watch.

3 The graph below shows the variation of angular velocity with time for a rotating body.



(a) Find the angular displacement θ covered in the first 3 seconds.

(b) Find the total angular displacement for the 6 seconds.

(c) Calculate the angular acceleration of the rotating body.

4 A wheel accelerates uniformly from rest. After 12 s the wheel is completing 100 revolutions per minute (r.p.m.)

(a) Convert 100 r.p.m. to its equivalent value in rad s^{-1}.

(b) Calculate the average angular acceleration of the wheel.

5 The angular velocity of a car engine's drive shaft is increased from 100 rad s^{-1} to 300 rad s^{-1} in 10 s.

(a) Calculate the angular acceleration of the drive shaft.

(b) Calculate the angular displacement during this time.

(c) A point on the rim of the drive shaft is at a radius of 0.12 m.

Calculate the distance covered by this point in the 10 s time interval.

6 Use calculus methods to derive the equations for angular motion. The method is very similar to that for linear motion.

Note: in the unit or course assessment you may be asked to derive the linear motion equations but **not** the angular motion equations.

TUTORIAL 3.0

Central force

1. (a) State the equation between radial acceleration and angular velocity.
   (b) State the units of angular and radial acceleration.
   (c) Explain the difference between angular and radial acceleration.

2. Derive the expression $\alpha = r\omega^2$ for the radial acceleration of an object.

3. The central force maintaining an object in a circular orbit is given by $F = mr\omega^2$.
   Sketch graphs showing the variations of:
   (a) central force with mass of the object
   (b) central force with radius of the object
   (c) central force with angular velocity of the object.

4. A piece of string has a breaking force of 56 N. This string is used to whirl a mass of 150 g in a horizontal circle.
   (a) The 150 g mass moves in a horizontal circle of radius 1.2 m. Calculate the maximum angular velocity of the mass.
   (b) The mass is rotated at 85 rpm. Find the maximum possible radius of the circular orbit.

5. A swing ball, on a cord of length 1.5 m, has a mass of 2.0 kg. After being hit by a bat, the ball moves in a horizontal circle of radius 0.50 m with a steady speed of 1.33 m s^{-1}.
   (a) Draw a sketch showing the path of the ball on the string.
   (b) Calculate the central acceleration of the ball.
   (c) Draw a sketch showing all the forces on the ball while moving in a horizontal circle. Determine the tension in the string.

6. A 3.0 kg mass is whirled in a vertical circle of radius 0.75 m at a steady speed of 8.0 m s^{-1}.
   (a) Calculate the tension in the string at the top of the circle.
   (b) Calculate the tension in the string at the bottom of the circle.

7. A hump backed bridge is in the form of a circular arc of radius 35 m.
   What is the greatest speed with which a car can cross the bridge without leaving the ground at its highest point?

8. (a) In a space flight simulator an astronaut is rotated at 20 rpm in a pod which is at the end of an arm of radius 5.0 m. Show that the central force on the astronaut is 2.2g.
   (b) What rotation rate would give a ‘simulated’ gravity of 3g?

9. Comment on the words centripetal and centrifugal with respect to angular motion.

TUTORIAL 3.1

Circular Motion

1 An Earth satellite is required to be in a circular orbit at a distance of 7.5×10^6 m from the centre of the Earth. The central force is due to the gravitational force. The acceleration due to the Earth's gravity at this point is 7.0 m s^{-2}

Find:

(a) the required satellite speed

(b) the period of revolution of the satellite.

2 What would be the period of rotation of the Earth about its axis if its speed of rotation increased to such an extent that an object at the equator became 'weightless'?

(Hint: equate mg to $\frac{mv^2}{r}$).

3 A sphere of mass 0.20 kg is rotating in a circular path at the end of a string 0.80 m long. The other end of the string is fixed. The period of the motion is 0.25 s.

(a) Calculate the tension in the string, which you may assume to be horizontal.

(b) In practice the string is not horizontal. Explain why this is so.

(c) Draw a force diagram for the sphere.
From this calculate the angle the string would make with the horizontal.

4 The moon takes 27.3 days (2.0×10^6 s) to complete one orbit of the Earth.
The distance between the centres of the Earth and Moon is 4.0×10^8 m.
Calculate the magnitude of the Moon's acceleration towards the Earth.

5 A ball of mass 2.0 kg is attached to a string 1.6 m long and is made to travel in a vertical circle. The ball passes its highest point with a speed of 5.0 m s^{-1}.

(a) What is the kinetic energy of the ball at its highest point?

(b) What is its potential energy when it is at the highest point (with reference to its lowest point)?

(c) What is its kinetic energy at the lowest point?

(d) What is its speed at the lowest point?

(e) What is the tension in the string at the highest and lowest points?

(f) What is the **least** speed the ball could have at the highest point in order to be able to complete a vertical circle at all?

6 An old humpback bridge has a radius of curvature of 20 m. What is the maximum speed at which a car can pass over this bridge if the car is not to leave the road surface?

7 (a) A pail of water is swinging in a vertical circle of radius 1.2 m, so that the water does not fall out. What is the minimum linear speed required for the pail of water.

(b) Convert this speed into an angular velocity.

8 An object of mass 0.20 kg is connected by a string to an object of half its mass. The smaller mass is rotating at a radius of 0.15 m on a table which has a frictionless surface. The larger mass is suspended through a hole in the middle of the table.
Calculate the number of revolutions per minute the smaller mass must make so that the larger mass is stationary.

Banking of a Track

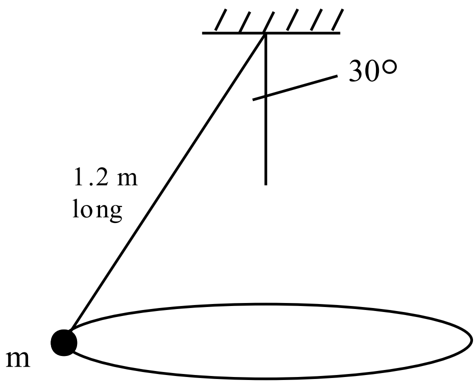
9 A circular track of radius 60 m is banked at angle □. A car is driven round the track at 20 m s^{-1}.

(a) Draw a diagram showing the forces acting on the car.

(b) Calculate the angle of banking required so that the car can travel round the track without relying on frictional forces (i.e. no side thrust supplied by friction on the track surface).

Conical Pendulum

10 A small object of mass m revolves in a horizontal circle at constant speed at the end of a string of length 1.2 m. As the object revolves, the string sweeps out the surface of a right circular cone.



The cone has semi-angle 30°.
Calculate:
(a) the period of the motion;
(b) the speed of the object.

[Hint: try resolving the tension in the string into horizontal and vertical components.]

TUTORIAL 4.0

Torque and moment of inertia

1. (a) State what is meant by the moment of a force.
   (b) Give two examples illustrating the moment of a force.

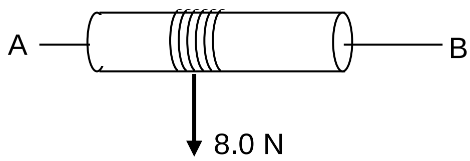
2. (a) State the equation between torque and tangential force.
   (b) State the equation between torque and angular acceleration.

3. The moment of inertia of an object depends on two quantities. State clearly the two quantities concerned.

4. The moment of inertia of a rod about an axis through its centre is different to the moment of inertia of the same rod about an axis through one end.
   Explain why this is so and justify which arrangement has the larger moment of inertia.

5. A wheel has very light spokes. The mass of the rim and tyre is 2.0 kg and the radius of the wheel is 0.80 m.
   Calculate the moment of inertia of the wheel. State any assumptions that you have made.

6. A cylindrical solid drum is free to rotate about an axis AB as shown below.



   The radius of the drum is 0.30 m. The moment of inertia of the drum about AB is 0.40 kg m^2. A rope of length 5.0 m is wound round the drum and pulled with a constant force of 8.0 N.
   (a) Calculate the torque on the drum.
   (b) Determine the angular acceleration of the drum.
   (c) Calculate the angular velocity of the drum just as the rope leaves the drum. You may assume that the drum starts from rest.

7. A hoop has a radius of 0.20 m and a mass of 0.25 kg.
   (a) What is the moment of inertia of the hoop?
   (b) What torque is required to give the hoop an acceleration of 5.0 rad s^{-2}?

8. A sphere has a moment of inertia of 0.40 MR^2 where M is the total mass of the sphere and R is the radius.
   (a) Calculate the moment of inertia of the Earth as it spins on its axis.
   State any assumptions made.
   (b) What is the tangential speed at the surface of the Earth at the Equator?

9. Two children are playing on a roundabout. One child, Anne, of mass 50 kg, stands on the roundabout 1.25 m from the axis of rotation. The other child, Robert, starts the roundabout by applying a constant torque of 200 N m at the rim for 3 s.
   When rotating there is a constant frictional torque of 25 N m acting on the roundabout. Robert stops pushing and the roundabout comes to rest.
   The moment of inertia of the roundabout *alone* is 500 kg m^{-2}.
   (a) Calculate the maximum angular velocity of the roundabout.
   (b) Find the time taken for the roundabout to come to rest.

TUTORIAL 5.0

Angular momentum and rotational kinetic energy

1. (a) State the law of conservation of angular momentum.
   (b) State the expression for the angular momentum of an object in terms of its moment of inertia.
   (c) State the equation for the rotational kinetic energy of a rigid object.

2. A bicycle wheel has a moment of inertia of 0.25 kg m^2 about its hub.
   Calculate the angular momentum of the wheel when rotating at 120 r.p.m.

3. A model aeroplane is flying in a horizontal circle at the end of a light wire. The mass of the aeroplane is 2.0 kg. The radius of the circular path is 20 m. The aeroplane makes 40 revolutions in one minute.
   (a) Calculate the linear velocity of the aeroplane.
   (b) Find the angular momentum of the aeroplane about the centre of the circle.
   (c) The wire suddenly breaks. What is the new angular momentum of the aeroplane about the centre of the circle?

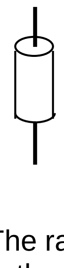
4. A shaft has a moment of inertia of 20 kg m^2 about its central axis. The shaft is rotating at 10 rpm. This shaft is locked onto another shaft, which is initially stationary. The second shaft has a moment of inertia of 30 kg m^2.
   (a) Find the angular momentum of the combination after the shafts are locked together.
   (b) What is the angular velocity of the combination after the shafts are locked together?

5. Which of the following are vector quantities:
   torque, moment of inertia, angular velocity, tangential force, angular acceleration, rotational kinetic energy, radius of a circular motion.

6. Two children are playing on a roundabout of mass 250 kg. The roundabout can be considered to be a solid disc of diameter 3.0 m. ($I_{disc} = \frac{1}{2} MR^2$)
   One child of mass 40 kg stands on the rim of the roundabout. The other child of mass 60 kg is positioned half way between the rim and the centre.
   (a) Calculate the total moment of inertia of the roundabout and children.
   (b) Determine the rotational kinetic energy of this system when it is rotating at 35 rpm.

7. A disc has a moment of inertia of 2.5 kg m^2. The disc is rotating at 2.0 rad s^{-1}.
   (a) Calculate the kinetic energy of the disc.
   (b) How much energy needs to be supplied to increase its angular velocity to 15 rad s^{-1}?

8. A solid cylinder and a hollow cylinder have the same mass and the same radius.
   (a) Which one has the larger moment of inertia about the central axis as shown opposite? You must justify your answer.
   (b) The cylinders do not have the same length. Does this affect your answer to part (a)? Again you must justify your answer.



9. A cylinder of mass 3.0 kg rolls down a slope without slipping. The radius R of the cylinder is 50 mm and its moment of inertia is $\frac{1}{2}MR^2$. The slope has a length of 0.30 m and is inclined at 40^o to the horizontal.
   (a) Calculate the loss in gravitational potential energy as the cylinder rolls from the top of the slope to the bottom of the slope.
   (b) Find the speed with which the cylinder reaches the bottom of the slope.

10. A turntable is rotating freely at 40 rpm about a vertical axis. A small mass of 50 g falls vertically onto the turntable and lands at a distance of 80 mm from the central axis. The rotation of the turntable is reduced to 33 rpm.
    Find the moment of inertia of the turntable.

11. A CD of mass 0.020 kg and diameter 120 mm is dropped onto a rotating turntable. The turntable has a moment of inertia about its axis of rotation of 5.0 x 10^{-3} kg m^2. The turntable was initially rotating at 3.0 rad s^{-1}.
    Determine the common angular velocity of the turntable and the CD.

12. A skater with her arms pulled in has a moment of inertia of 1.5 kg m^2 about a vertical axis through the centre of her body. With her arms outstretched the moment of inertia is increased to 10 kg m^2.
    With her arms pulled in, the skater is spinning at 30 rad s^{-1}. The skater then extends her arms.
    (a) Calculate her final angular speed.
    (b) Find the change in kinetic energy.
    (c) Explain why there is a change in kinetic energy.

13. A skater is spinning at 3.0 rad s^{-1} with her arms and one leg outstretched.
    The angular speed is increased to 25 rad s^{-1} when she draws her arms and leg in.
    (a) Explain why this movement of her arms and leg affects the rotational speed.
    (b) Her moment of inertia about her spin axis is 5.0 kg m^2 with her arms and leg outstretched. Calculate her moment of inertia when her arms and leg are drawn in.

14. A roundabout has a moment of inertia of 300 kg m^2 about its axis of rotation. Three children, each of mass 20 kg, stand 2.0 m from the centre of the stationary roundabout. They all start to run round the roundabout in the same direction until they reach a speed of 3.0 m s^{-1} relative to the roundabout.
    Calculate the angular velocity of the roundabout.

15. A disc is rotating at 100 rpm in a horizontal plane about a vertical axis. A small piece of plasticene is dropped vertically onto the disc and sticks at a position 50 mm from the centre of the disc. The plasticene has a mass of 20 g. The disc is slowed to 75 rpm. Calculate the moment of inertia of the disc.

16. The radius of a spherical neutron star is 20 km. The star rotates at 0.5 rpm.
    (a) Calculate the velocity of a point on the equator of the star.
    (b) The mass of the neutron star is the same as the mass of the sun.
    What is the density of the neutron star?
    (c) The radius of a neutron is about 10^{-15} m. Estimate the average spacing of the neutrons in the neutron star.

TUTORIAL 5.1

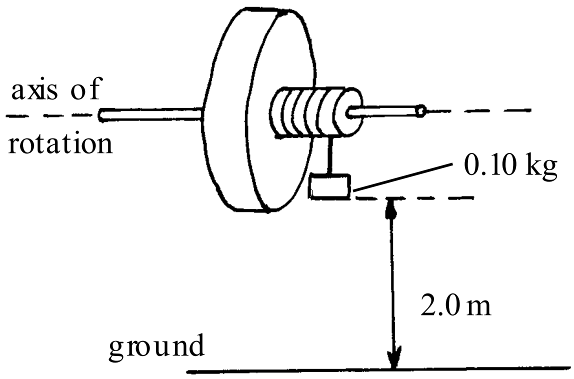
Torque, Moments of Inertia and Angular Momentum

1 A flywheel has a moment of inertia of 1.2 kg m^2. The flywheel is acted on by a torque of magnitude 0.80 N m.

(a) Calculate the angular acceleration produced.

(b) The torque acts for 5.0 s and the flywheel starts from rest. Calculate the angular velocity at the end of the 5.0 s.

2 A mass of 0.10 kg is hung from the axle of a flywheel as shown below. The mass is released from a height of 2.0 m above ground level.



The following results were obtained in the experiment:

time for mass to fall to the ground t = 8.0 s

radius of axle R = 0.10 m.

(a) By energy considerations, show that the final speed of the flywheel is given by $v = \sqrt{\frac{3.92}{0.1 + 100I}}$ where *I* is the moment of inertia of the flywheel. Friction effects are ignored.

(b) Calculate the moment of inertia of the flywheel.

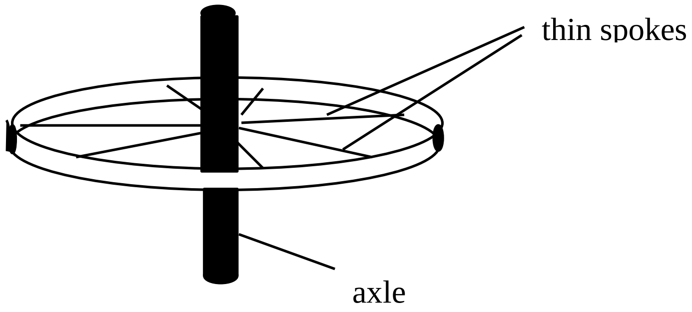
3 A heavy drum has a moment of inertia of 2.0 kg m^2. It is rotating freely at 10 rev s^{-1} and has a radius of 0.50 m. A constant frictional force of 5.0 N is then exerted at the rim of the drum.

(a) Calculate the time taken for the drum to come to rest.

(b) Calculate the angular displacement in this time.

(c) Hence calculate the heat generated in the braking action.

4 A cycle wheel is mounted so that it can rotate horizontally as shown.
Data on wheel: radius of wheel = 0.50 m, mass of wheel = 2.0 kg.



(a) Calculate the moment of inertia of the wheel system. State any assumptions you make.

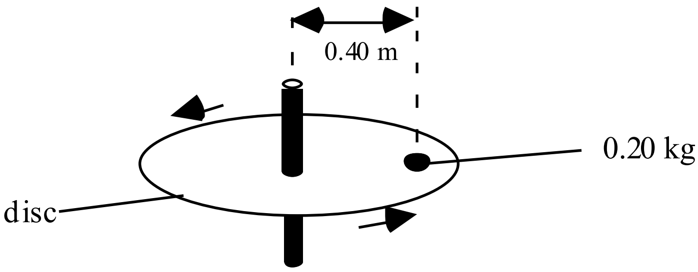
(b) A constant driving force of 20 N is applied to the rim of the wheel.

(i) Calculate the magnitude of the driving torque on the wheel.
(ii) Calculate the angular acceleration of the wheel.

(c) After a period of 5.0 s, calculate:

(i) the angular displacement,
(ii) the angular momentum of the wheel, and
(iii) the kinetic energy of the wheel.

5 A very light but strong disc is mounted on a free turning bearing as shown below.



A mass of 0.20 kg is placed at a radius of 0.40 m and the arrangement is set rotating at 1.0 rev s^{-1}.
(The moment of inertia of the disc can be considered to be negligible.)

(a) Calculate the angular momentum of the 0.20 kg mass.

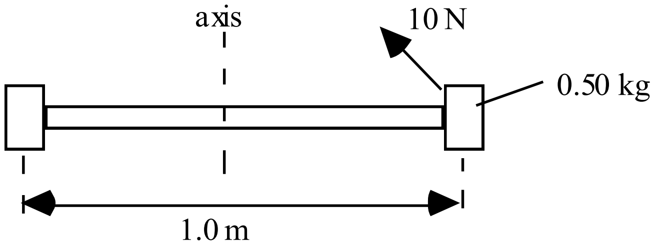
(b) Calculate the kinetic energy of the mass.

(c) The mass is pushed quickly into a radius of 0.20 m.

By applying the principle of conservation of angular momentum, calculate the new angular velocity of the mass in rad s^{-1}.

(d) Find the new kinetic energy of the mass and account for any difference.

6 A uniform metal rod has a mass, M, of 1.2 kg and a length, L, of 1.0 m. Clamped to each end of the rod is a mass of 0.50 kg as shown below.



(a) Calculate an approximate value for the moment of inertia of the complete arrangement about the central axis as shown. Assume that $I_{rod} = \frac{1}{12} ML^2$ about this axis.

(b) The arrangement is set rotating by a force of 10 N as shown in the diagram. The force acts at a tangent to the radius.

(i) Calculate the applied torque.

(ii) Hence find the maximum angular acceleration. You may assume that the force of friction is negligible.

(iii) Calculate the kinetic energy of the arrangement 4.0 s after it is set rotating.

7 An unloaded flywheel, which has a moment of inertia of 1.5 kg m^2, is driven by an electric motor. The flywheel is rotating with a constant angular velocity of 52 rad s^{-1}. The driving torque, of 7.7 N m, supplied by the motor is now removed.

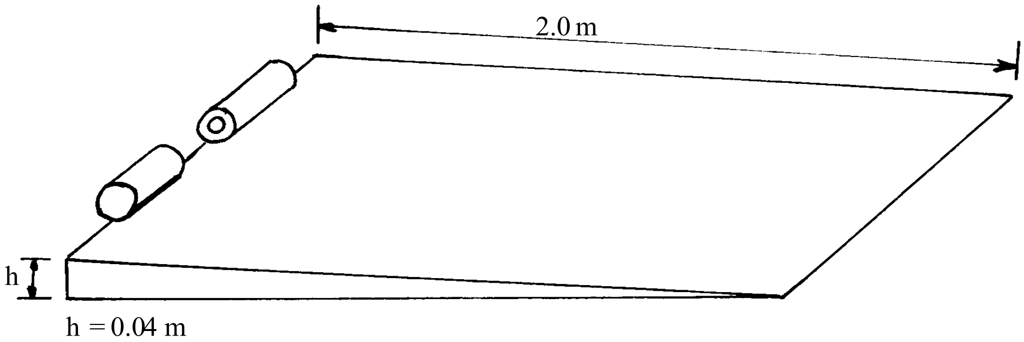
How long will it take for the flywheel to come to rest.
You may assume that the frictional torque remains constant?

8 A solid aluminium cylinder and a hollow steel cylinder have the same mass and radius. The two cylinders are released together at the top of a slope.

(a) Which of the two cylinders will reach the bottom first?

(b) Explain your answer to part (a).

9 A solid cylinder and a hollow cylinder each having the same mass M and same outer radius R, are released at the same instant at the top of a slope 2.0 m long as shown below.
The height of the slope is 0.04 m.



end-on view of the cylinders

$M = 10\ \text{kg},\ R = 0.10\ \text{m}$	$M = 10\ \text{kg},\ R = 0.10\ \text{m},\ r = 0.05\ \text{m}$
$I = \frac{1}{2} M R^2$	$I = \frac{1}{2} M (R^2 + r^2)$

It is observed that one of the cylinders reaches the bottom of the slope before the other.

(a) Using the expressions given above, show that the moments of inertia for the cylinders are as follows:

(i) solid cylinder; $I = 0.05\ \text{kg m}^2$ (ii) hollow cylinder; $I = 0.0625\ \text{kg m}^2$.

(b) By energy considerations, show that the linear velocity of any cylinder at the bottom of the slope is given by:

$$v = \sqrt{\frac{2gh}{[1 + \frac{I}{MR^2}]}}.$$

(c) Using the expression in (b) above, calculate the velocities of the two cylinders at the bottom of the slope and hence show that one of the cylinders arrives at the bottom of the slope 0.23 s ahead of the other.