Wallace Hall Academy
Physics Department

Advanced Higher Physics

Waves

Solutions

TUTORIAL 1.0

Simple harmonic motion

1. (a) The unbalanced force is proportional to the displacement, and acts in the opposite direction.

   (b) A mass oscillating at the end of a spring, a pendulum with small amplitude, a hack saw blade fixed at on end and vibrating at the other.

2. (a) $\frac{d^2y}{dt^2} = -\omega^2 y$

   (b) (i)

$$\boxed{y = a\cos\omega t \quad \text{if } y = 0 \text{ at } t = 0 \qquad \text{and} \qquad y = a\sin\omega t \quad \text{if } y = a \text{ at } t = 0}$$

Acceleration

Differentiating $\frac{dy}{dt} = \frac{d}{dt}(a\cos\omega t)$

$= -a\omega\sin\omega t$

Differentiating again $\frac{d^2y}{dt^2} = -a\omega^2\cos\omega t$

but $y = a\cos\omega t$ $\frac{d^2y}{dt^2} = -\omega^2 y$

$\frac{dy}{dt} = \frac{d}{dt}(a\sin\omega t)$

$= a\omega\cos\omega t$

$\frac{d^2y}{dt^2} = -a\omega^2\sin\omega t$

$\frac{d^2y}{dt^2} = -\omega^2 y$ $(y = a\sin\omega t)$

(ii) $y = a$ at $t = 0$ and $y = 0$ at $t = 0$.

(c)

Velocity

$v = \frac{dy}{dt} = -a\omega\sin\omega t$

$v^2 = a^2\omega^2\sin^2\omega t$ and $y^2 = a^2\cos^2\omega t$

$\sin^2\omega t + \cos^2\omega t = 1$

Thus $\frac{v^2}{a^2\omega^2} + \frac{y^2}{a^2} = 1$

$v^2 = \omega^2(a^2 - y^2)$

Thus $\boxed{v = \pm\omega\sqrt{a^2 - y^2}}$

$v = \frac{dy}{dt} = a\omega\cos\omega t$

$v^2 = a^2\omega^2\cos^2\omega t$ and $y^2 = a^2\sin^2\omega t$

$\cos^2\omega t + \sin^2\omega t = 1$

Thus $\frac{v^2}{a^2\omega^2} + \frac{y^2}{a^2} = 1$

$v^2 = \omega^2(a^2 - y^2)$

Thus $\boxed{v = \pm\omega\sqrt{a^2 - y^2}}$

3. (a) acceleration = $-\omega^2 y$ acceleration at centre = 0 $(y = 0)$

   acceleration at extremities = $\pm\ 4\pi^2 \text{ x } 5^2 \text{ x } 0.04 = \pm 39 \text{ m s}^{-2}$ $(\omega = 2\pi f)$

   (b) velocity = $\pm\ \omega\sqrt{a^2 - y^2}$

   velocity at extremities = 0

   velocity at centre = $\pm\ \omega a = \pm 2\pi \text{ x } 5 \text{ x } 0.04 = \pm 1.3 \text{ m s}^{-1}$

   (c) acceleration = $\pm 4\pi^2 \text{ x } 5^2 \text{ x } 0.02 = \pm 20 \text{ m s}^{-2}$

   velocity = $\pm 2\pi \text{ x } 5 \text{ x} \sqrt{0.04^2 - 0.02^2} = \pm 1.1 \text{ m s}^{-1}$

4. Acceleration of oscillations must be numerically less that g = 9.8 m s^{-2}

$$9.8 = \omega^2 a = 2\pi \text{ x } \frac{1}{0.1} \text{ x a} \qquad \text{giving a} = 0.16 \text{ m}$$

5 (a)

Kinetic energy equation for the particle

$$E_k = \frac{1}{2} \text{ m } v^2 \qquad = \frac{1}{2} \text{ m } [\pm \omega \sqrt{a^2 - y^2}\ ]^2$$

$$\boxed{E_k = \frac{1}{2} \text{m } \omega^2 (a^2 - y^2)}$$

Potential energy equation for the particle

When at position O the potential energy is zero, (with reference to the equilibrium position) and the **kinetic energy is a maximum**.

The kinetic energy is a maximum when y = 0: $E_{kmax} = \frac{1}{2} \text{ m } \omega^2 a^2$

At point O total energy $E = E_k + E_p = \frac{1}{2} \text{ m } \omega^2 a^2 + 0$

$$E = \frac{1}{2} \text{ m } \omega^2 a^2 \qquad \text{or} \qquad E = \frac{1}{2} \text{ k } a^2 \quad \text{because } \omega^2 = \frac{k}{m}$$

The **total energy E** is the **same at all points** in the motion.

Thus for any point on the swing: as above $E = E_k + E_p$

$$\frac{1}{2} \text{ m } \omega^2 a^2 = \frac{1}{2} \text{ m } \omega^2 (a^2 - y^2) + E_p$$

$$\boxed{E_p = \frac{1}{2} \text{m } \omega^2 y^2}$$

(b) $E_k = ½ \text{ m}\omega^2(a^2 - y^2)$ $E_p = ½ \text{ m}\omega^2 y^2$

(i) E_k(max) = ½ x 0.2 x (2π x 0.5)^2 x 0.1^2 = 9.9 x 10^{-3} J at the central position

(ii) E_k(min) = 0 when y = a, at the extreme position

(iii) E_p(max) = ½ x 0.2 x (2π x 0.5)^2 x 0.1^2 = 9.9 x 10^{-3} J at the extreme position

(iv) E_k(min) = 0 at the central position when y = 0

(v) Sum of E_p and E_k at any point is always the same and equal to 9.9 x 10^{-3} J

6. (a) 0.44 mm

(b) 28 = 2πf f = 4.5 Hz

(c) period T = 1/f = 0.22 s

(d) $0.2 = 0.44\sin 28t$ hence $\sin 28t = \frac{0.2}{0.44} = 0.4545$

28t = 0.472 (remember to work in radians) giving t = 0.017 s

7. (a) Reduces the amplitude of the oscillations

(b) When a system is critically damped the frictional resistance is just enough to prevent any oscillations occurring. The time taken for the displacement to become zero is a minimum.

When a system is overdamped the frictional resistance is large and the system takes a long time to come to rest.

(c) Critical damping for the needle on a meter.

Critical damping for car shock absorbers

TUTORIAL 1.1

Simple harmonic motion

1 (a) $y = 4 \cos 4\pi t$ compare with $y = a \cos \omega t$

thus $a = y_{max} = \underline{\underline{4}}$ cm

(b) $\omega = 4\pi$ and $\omega = 2\pi f$

$4\pi = 2\pi f$ giving $f = \underline{\underline{2}}$ Hz

Also $f = \frac{1}{T}$ thus $T = \underline{\underline{0.5}}$ s

(c) (i) when $t = 0$ s $y = 4 \cos 0 = \underline{\underline{4}}$ cm

(ii) when $t = 1.5$ s $y = 4 \cos (4\pi \times 1.5)$ (Remember angle in radians)

$= \underline{\underline{4}}$ cm

2 (a) $f = 40$ Hz thus $T = \frac{1}{f} = \frac{1}{40} = \underline{\underline{0.025}}$ s

(b) $y = a \cos \omega t = a \cos 2\pi f t = \underline{\underline{0.05 \cos 80\pi t}}$

(c) (i) acceleration $= - \omega^2 y$

at mid-point $y = 0$ thus acceleration $= \underline{\underline{0}}$ m s^{-2}

at max amplitude $y = 0.05$ m

acceleration $= - (2\pi \times 40)^2 \times 0.05$

$= - \underline{\underline{3.2 \times 10^3}}$ m s^{-2} directed towards the midpoint

(ii) $v_{max} = \pm \omega a = \pm 2\pi \times 40 \times 0.5$

$= \underline{\underline{\pm 12.6}}$ m s^{-1} this occurs at the midpoint when $y = 0$.

3 (a) $y = a \cos \omega t$ (at $t = 0$, $y = a$)

$= 0.12 \cos \frac{2\pi}{1.5} \times 0.4$ (remember ωt in radians)

$= \underline{\underline{-0.0125}}$ m (or $y = -0.013$ m to 2 sig figs)

The position of object is 0.0125 m on the **opposite** side of the equilibrium position to $y = a$ at $t = 0$.

(b) Use $F = m \times$ acceleration and acceleration $= - \omega^2 y$

$= m \times - \omega^2 y$

$= - 0.5 \times (\frac{2\pi}{1.5})^2 \times 0.0125$

$= \underline{\underline{0.11}}$ N

The force is acting in the positive direction, towards the equilibrium position.

(c) $y = - 0.06$ m and $- 0.06 = 0.12 \times \cos (\frac{2\pi}{1.5} \times t)$

$\cos (\frac{2\pi}{1.5} \times t) = - \frac{0.06}{0.12}$ and $\frac{2\pi}{1.5} \times t = \cos^{-1} (- \frac{0.06}{0.12})$

$t = \frac{1.5}{2\pi} \cos^{-1} (- \frac{0.06}{0.12})$

$t = \underline{\underline{0.5}}$ s

4 (a) $y = a \sin \omega t$ and $y_{max} = a$

$y = 2.0 \sin (3.22 \times 10^3 t)$ thus $y_{max} = \underline{\underline{2.0}}$ mm

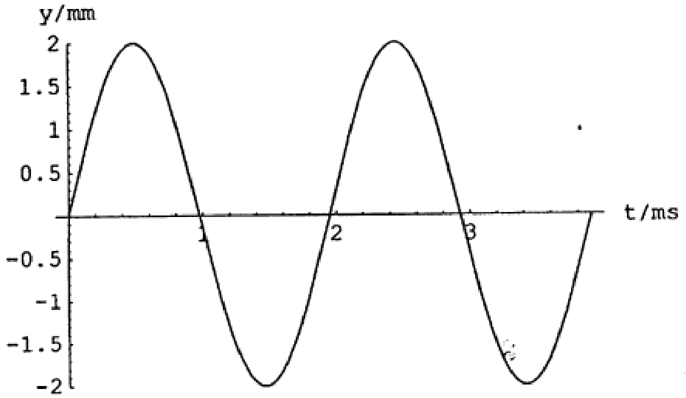
$\omega = 2\pi f$ and $\omega = 3.22 \times 10^3$

$$f = \frac{3.22 \times 10^3}{2\pi} = \underline{\underline{512}} \text{ Hz}$$

(b) $accn_{max} = - \omega^2 y_{max} = - (2\pi \times 512)^2 \times 2.0 \times 10^{-3}$

$= \underline{\underline{2.07 \times 10^4}} \text{ m s}^{-2}$

(c) $T = \frac{1}{512} = 1.95 \times 10^{-3}$ s and $y = 0$ when $t = 0$



(d) The period of any SHM is constant even although the amplitude is decreasing.

5 Acceleration will have to be greater than 10 m s^{-2} for this condition to occur.

Use $accn_{max} = - \omega^2 y_{max}$ and $\omega = 2\pi f$

$= (2 \times \pi \times 40)^2 \; y_{max}$

$$y_{max} = \frac{10}{(2 \times \pi \times 40)^2} = \underline{\underline{1.58 \times 10^{-4}}} \text{ m}$$

6 (a) $k = \frac{\text{force}}{\text{extension}} = \frac{1.2 \times 9.8}{0.10} = \underline{\underline{118}} \text{ N m}^{-1}$

(b) (i) amplitude $= \underline{\underline{0.08}}$ m

(ii) $\omega^2 = \frac{k}{m} = \frac{118}{1.2}$ and $T = \frac{2\pi}{\omega}$

$T = 2\pi \sqrt{\frac{1.2}{118}} = \underline{\underline{0.63}}$ s and $f = \frac{1}{T} = \underline{\underline{1.6}}$ Hz

(iii) $v_{max} = \pm \omega a = \pm \sqrt{\frac{118}{1.2}} \times 0.08 = \underline{\underline{\pm 0.79}} \text{ m s}^{-1}$

Total energy $= \frac{1}{2} m \omega^2 a^2 = \frac{1}{2} \times 1.2 \times \frac{118}{1.2} \times 0.08^2$

$= \underline{\underline{0.38}}$ J

7 First use the conservation of linear momentum to find the velocity just after the dart embeds.

In the absence of external forces;

$$\text{total momentum before} = \text{total momentum after}$$

$$mv_1 = (m + M) \text{ x } v_2$$

$$0.060 \text{ x } 120 = (5.0 + 0.06) \text{ x } v_2$$

$$v_2 = \frac{0.060 \text{ x } 120}{5.06} = 1.42 \text{ m s}^{-1}$$

This is the maximum velocity, v_{max}

(a) $v_{max} = \omega a$ and $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{450}{5.06}}$

$$a = \frac{v_{max}}{\omega} = 1.42\sqrt{\frac{5.06}{450}} = \underline{\underline{0.15}} \text{ m}$$

An alternative solution can be found from an analysis of the energy.

$$E_{tot} \text{ (system)} = \frac{1}{2} \text{ m } v_{max}{}^2 = \frac{1}{2} \text{ x } 5.06 \text{ x } (1.42)^2$$

$$= 5.10 \text{ J}$$

but $E_p = \frac{1}{2} k a^2$ when $v = 0$ i.e. $E_k \text{ -> } E_p$

thus $a^2 = \frac{2 \text{ x } E_{tot}}{k}$ and $a = \sqrt{\frac{2 \text{ x } 5.10}{450}} = \underline{\underline{0.15}} \text{ m}$

(b) E_k (total of system) = 5.1 J (see above $E_{tot} = \frac{1}{2} \text{ m } v_{max}{}^2$)

$$E_k \text{ (of dart)} = \frac{1}{2} \text{ m } v^2 = \frac{1}{2} \text{ x } 0.06 \text{ x } (120)^2$$

$$= 432 \text{ J}$$

percentage of energy in oscillating system $= \frac{5.1}{432} \text{ x } \frac{100}{1} = \underline{\underline{1.2}}$ %

8 An oscillating system will eventually come to rest if there is no driving force. The amplitude of the oscillations decrease due to the presence of friction. This decrease in the amplitude is called damping.

Critical damping occurs when the frictional resistance is just sufficient to prevent any oscillations past the rest position. Critical damping occurs when an oscillating system comes to rest in the shortest possible time.

It is worth noting that in the process of damping the energy of the system ends up as heat energy which is transferred to the surroundings.

TUTORIAL 2.0

Waves

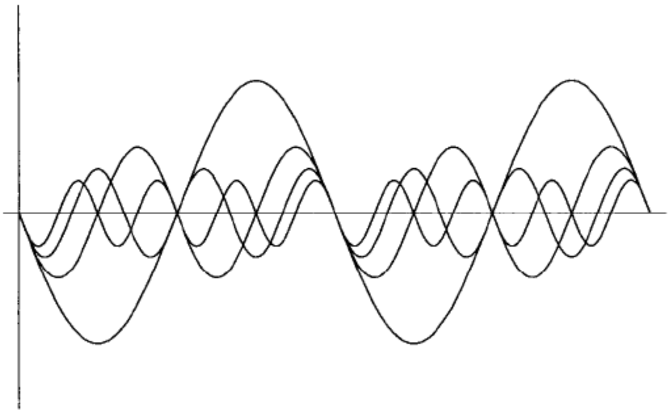
1. (a) intensity $\propto$ (amplitude)^2
   (b) an increase of 81 times

2. Any waveform can be represented by a series of sine or cosine expressions.

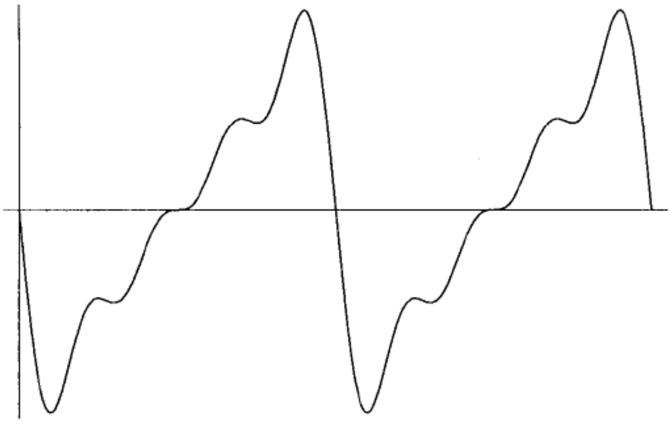
A **periodic** wave is a wave which repeats itself at regular intervals. All periodic waveforms can be described by a mathematical series of sine or cosine waves, known as a Fourier Series. For example a saw tooth wave can be expressed as a series of individual sine waves.

$$y(t) = -\frac{1}{\pi}\sin\omega t - \frac{1}{2\pi}\sin 2\omega t - \frac{1}{3\pi}\sin 3\omega t - \ldots\ldots\ldots\ldots$$

The graph below shows the first four terms of this expression.



When all these terms are superimposed (added together) the graph below is obtained. Notice that this is tending to the sawtooth waveform. If more terms are included it will have a better saw tooth form.



3. (a) y – displacement in transverse direction
a – amplitude f – frequency t – time
x – distance of a particle from the origin λ - wavelength

(b) (i) 0.06 m
(ii) 75 Hz [using $2\pi f = \pi \times 150$]
(iii) 13 ms [using $T = 1/f$]
(iv) 5 m [using $2\pi/\lambda = \pi \times 0.40$
(v) 375 m s^{-1} [using $v = \lambda f$]

4. (a) amplitude, frequency, period
(b) no, at time t = 0 the displacements are different.

5. (a) A stationary wave does not travel to the left or the right, but particle displacements do still take place. The particle displacements increase or decrease in unison. In some places there are maximum amplitudes in other places zero amplitude and no vibration.
(b) A node is a position of zero amplitude.
An antinode is a position of maximum amplitude.

TUTORIAL 2.1

Waves

1 (a) $y = 3 \sin 2\pi (10t - 0.2x)$ is compared with $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$a = \underline{\underline{3.0}}$ cm

(b) $f = \underline{\underline{10}}$ Hz

(c) $\frac{1}{\lambda} = 0.2$ thus $\lambda = \frac{1}{0.2} = \underline{\underline{5.0}}$ cm

(d) $v = f\lambda = 10 \times 5.0 = \underline{\underline{50}}$ cm s^{-1}

2 $y = a \sin 2\pi (ft - \frac{x}{\lambda})$ thus $y = 0.30 \sin 2\pi (20t - \frac{x}{0.5})$

3 $y = 0.20 \sin (220\pi t - 30\pi x)$ rewrite in the form $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$y = 0.20 \sin 2\pi (110t - 15x)$

assuming v remains the same, doubling f halves λ

thus $y = 0.40 \sin 2\pi (220t + 30x)$

4 $y = 0.04 \sin [2\pi(\frac{t}{0.04} - \frac{x}{2.0})]$

(a) $y_{max} = \underline{\underline{0.04}}$ m

(b) compare with $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$\lambda = \underline{\underline{2.0}}$ m

(c) $f = \frac{1}{0.04} = \underline{\underline{25}}$ Hz

(d) The movement of a particle will be Simple Harmonic with a maximum amplitude of 0.04 m. The particle will move in a direction perpendicular to the wave direction along the string.

5 (a) $y = 0.01 \sin \pi (2.0t - 0.01x)$

$= 0.01 \sin 2\pi (t - \frac{0.01}{2} x)$

thus $f = 1.0$ Hz and $\lambda = \frac{2}{0.01} = 200$ m

$v = f\lambda = 1.0 \times 200 = \underline{\underline{200}}$ m s^{-1}

(b) at $x = 0$ $y = 0.01 \sin \pi (2.0t)$

$= 0.01 \sin 2\pi t$

$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.01 \sin 2\pi t) = [0.01 \cos 2\pi t]\, 2\pi$

thus $v_{ymax} = 0.01 \times 1.0 \times 2\pi = \underline{\underline{0.063}}$ m s^{-1}

6 (a) $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

substitute $f = \frac{1}{T}$ giving $y = a \sin 2\pi (\frac{t}{T} - \frac{x}{\lambda})$

(b) substitute $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$

$y = a \sin 2\pi (ft - \frac{x}{\lambda}) = y = a \sin (2\pi ft - \frac{2\pi x}{\lambda})$

$y = a \sin (\omega t - kx)$

(c) substitute $\lambda = \frac{v}{f}$ into $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$y = a \sin 2\pi (ft - \frac{x}{\lambda}) = y = a \sin 2\pi (ft - \frac{xf}{v})$

$y = a \sin 2\pi f(t - \frac{x}{v})$

(d) substitute $f = \frac{v}{\lambda}$ into $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$y = a \sin 2\pi (\frac{vt}{\lambda} - \frac{x}{\lambda})$

$y = a \sin \frac{2\pi}{\lambda} (vt - x)$

7 (a) A phase difference of 2π occurs in one wavelength $\lambda = \frac{v}{f} = \frac{350}{500} = 0.70$ m

thus a phase difference of $\frac{\pi}{3}$ occurs in $\frac{1}{6}\lambda$

the two points are separated by $\frac{0.70}{6} = \underline{\underline{0.12}}$ m

(Alternatively use $\phi = \frac{2\pi x}{\lambda}$)

(b) 2π phase difference occurs in one period (T)

$T = \frac{1}{f} = \frac{1}{500} = 0.002$ s

0.002 s is equivalent to a phase difference of 2π
0.001 s is equivalent to a phase difference of π

8 (a) $\lambda = \frac{v}{f} = \frac{30}{250} = 0.12$ m

phase difference $= \frac{2\pi x}{\lambda} = 2\pi \times \frac{10}{12} = \underline{\underline{1.67\,\pi}}$ radians $= \underline{\underline{5.24}}$ radians

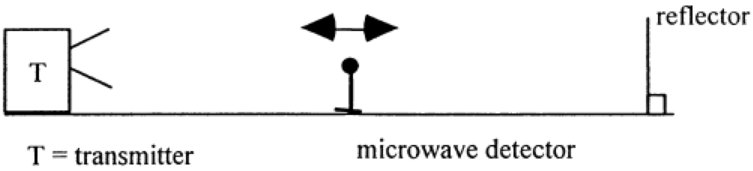
(b) $y = a \sin 2\pi (ft - \frac{x}{\lambda}) = 0.03 \sin 2\pi (250t - \frac{x}{0.12})$

(c) distance between nodes $= \frac{\lambda}{2} = \frac{0.12}{2} = \underline{\underline{0.06}}$ m

9 (a) A travelling wave is a wave which moves through a material transferring energy in the direction of travel. All particles of the material which transmits the energy perform Simple Harmonic Motion.

A stationary wave has parts of the material at rest, the nodes, and energy does not travel along the material. Energy is effectively trapped between the nodes. The particles between the nodes vibrate in phase with SHM but have different amplitudes.

(b) Wavelength of microwaves



Set up the apparatus as above. Move the microwave detector between transmitter and reflector and note that nodes are detected.
Measure the distance, d, between the first and eleventh node for example.

The distance between adjacent nodes $= \frac{\lambda}{2}$

Thus distance between first and eleventh nodes is 5λ giving $\lambda = \frac{d}{5}$.

10 (a) (i) The microphone has been moved a distance of $\frac{\lambda}{2}$.

thus $\lambda = 2 \times 0.24 = \underline{\underline{0.48}}$ m

(ii) $v = f\lambda = 700 \times 0.48 = 336$ m s^{-1}

velocity of sound $= \underline{\underline{340}}$ m s^{-1} (2 sig figs)

(b) As the listener walks across the room he will hear alternately quiet and loud sounds of the same frequency. This is because there are two sources of sound producing coherent waves and as they overlap, constructive and destructive interference takes place. Constructive interference will be positions of loud sound and destructive interference positions of quiet.

At constructive interference the path difference is $n\lambda$, the waves are in phase, the amplitude is bigger so there is more energy. At destructive interference the path difference is $(n + ½)\ \lambda$, the waves are completely out of phase, the amplitude is zero so there is no energy. The quiet patches are not completely silent because there is a degree of reflection of the sound from the walls in the room.

TUTORIAL 3.0

Interference – division of amplitude

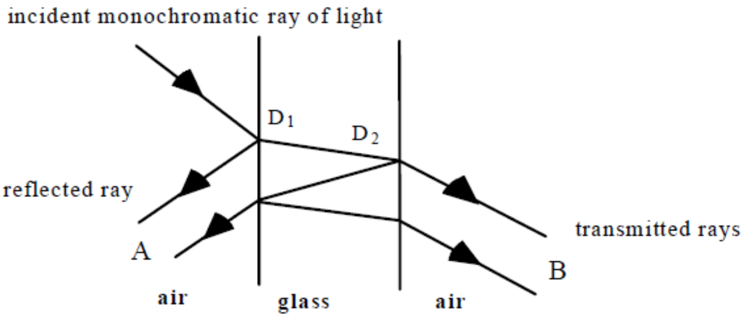
1. (a) Constant phase difference.
   (b) Light is produced when electrons, which have been excited, return to a lower energy state. This is a random process in that two separate sources will not emit light beams which have a constant phase difference, even if they have the same frequency.
   (c) yes, both loudspeakers are driven by the same single source. Any change in phase from the single source occurs simultaneously at the loudspeakers.

2. (a) Optical path difference = geometric path difference x refractive index
   (b) phase difference = $\frac{2\pi}{\lambda}$ x optical path difference
   (c) (i) optical path AB = (80 x 1.5) + (150 – 80) = 190 nm
   (ii) optical path difference = 190 - 150 = 40 nm

   $$\text{phase difference} = \frac{2\pi}{700 \times 10^{-9}} \times 40 \times 10^{-9} = 0.36 \text{ radians}$$

3 (a) π radians (or 180°)
   (b) None, the water has a smaller refractive index than the glass.

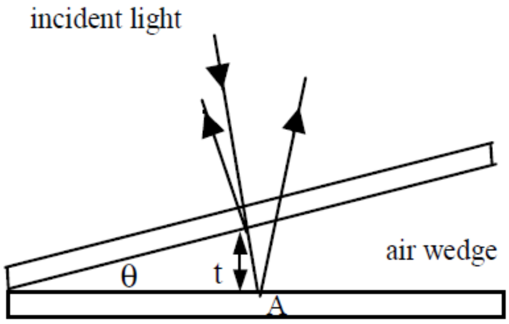
4. (a) Light incident on the film. The amplitude of the ray is divided. The light is partially reflected and partially refracted at D_1.
   The reflected rays at A have a different path difference so will interfere when brought together, similarly for the transmitted rays at B.



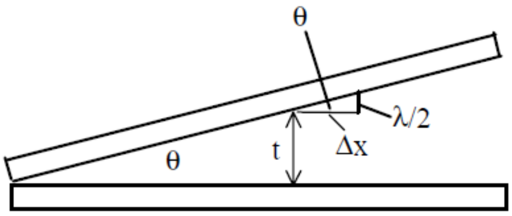
   (b) (i) 2nt = mλ for destructive interference
   (ii) When t increases the value of λ needed to produce constructive interference will increase. The colour of the pattern will move towards the red end of the spectrum.

5 (a)

Two glass slides are arranged as shown below.
Division of amplitude takes place at the *lower* surface of the top glass slide.



Enlarged view showing the geometry



When viewed from above the optical path difference $= 2t$
There is a phase difference of π on reflection at A. Hence the condition for a dark fringe is $2t = m\lambda$ assuming an air wedge.

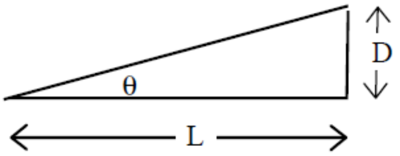
For the next dark fringe t increases by $\frac{\lambda}{2}$ (see right hand sketch above).

Thus the spacing of fringes, Δx, is such that $\tan\theta = \frac{\lambda}{2\,\Delta x}$ giving

$$\boxed{\Delta x = \frac{\lambda}{2\tan\theta}}$$

For a wedge of length L and spacing D

$\tan\theta = \frac{D}{L}$.



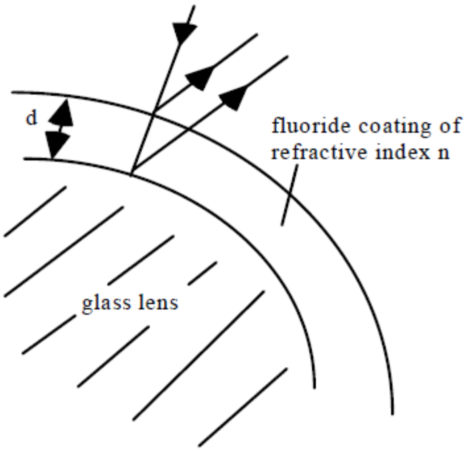
The fringe spacing is given by $\boxed{\Delta x = \frac{\lambda L}{2D}}$ where λ is the wavelength of light in air.

(b) $\Delta x = \frac{\lambda L}{2D} = \frac{650 \times 10^{-9} \times 100 \times 10^{-3}}{2 \times 30 \times 10^{-6}} = 1.1$ mm

(c) The thickness of the paper has increased. The wavelength and length of the plates are constant.

6 (a)

Good quality lenses in a camera reflect very little light and appear dark or slightly purple. A thin coating of a fluoride salt such as magnesium fluoride on the surface of the lens allows the majority of the light falling on the lens to pass through.
The refractive index, n, of the coating is chosen such that $1 < n < n_{glass}$.



Notice that there is a phase change of π at both the first and second surfaces.

For cancellation of reflected light:

optical path difference $= \frac{\lambda}{2}$.

Optical path in fluoride = 2nd

thus $2nd = \frac{\lambda}{2}$ and

$$\boxed{d = \frac{\lambda}{4n}}$$

(b) $d = \frac{\lambda}{4n} = 8.8 \text{ x } 10^{-8}$ m

(c) The non-reflective coating will only give complete cancellation for one particular wavelength. For a coating giving cancellation for green light, the blue and red would be partially reflected and the lens would appear purple.

TUTORIAL 3.1

Interference – division of amplitude

Interference - division of amplitude

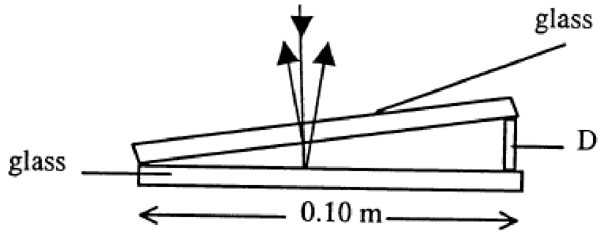
1 (a) Coherent sources must have a constant phase relationship. The two or more sources will come from the same original source.

(b) When we try to produce an interference pattern from two separate light sources it does not work because light is produced in small wave packets and not as a continuous wave. This is not the case for sound waves. We can have two separate loudspeakers, connected to the same signal generator, and produce an interference pattern.

2 (a) Division of amplitude involves splitting a single beam into two beams by producing a reflected beam and a transmitted beam at a surface between two materials of different refractive index. They may be multiple reflections and transmissions.

(b) An extended beam of light can be used because the beam is sub-divided by reflection and transmission at a surface. Hence there will always be a fixed phase relation between the sub-divided parts.

3 (a) Fringes are formed when reflections from the bottom surface of the top glass plate interfere with the beam transmitted through the top plate and reflected from the top surface of the bottom plate, see sketch below.



(b) $\tan\theta = \dfrac{\lambda}{2\Delta x}$ where λ = wavelength and Δx = fringe spacing

$= \dfrac{6.9 \times 10^{-7}}{2 \times 1.2 \times 10^{-3}}$

from sketch $D = \tan\theta \times 0.10 = \dfrac{6.9 \times 10^{-7}}{2 \times 1.2 \times 10^{-3}} \times 0.10$ D = thickness of foil

$= \underline{\underline{2.9 \times 10^{-5}}}$ m

(c) new value of $D = 2.9 \times 10^{-5} \times 1.1 = 3.2 \times 10^{-5}$ m i.e. 10% bigger

new $\tan\theta = \dfrac{3.2 \times 10^{-5}}{0.10}$

new $\Delta x = \dfrac{\lambda}{2\tan\theta} = \dfrac{6.9 \times 10^{-7} \times 0.10}{2 \times 3.2 \times 10^{-5}}$

$= \underline{\underline{1.1 \times 10^{-3}}}$ m

4 (a) For cancellation of reflected light: optical path in fluoride = $\frac{\lambda}{2}$

thus $2nd = \frac{\lambda}{2}$ where n = refractive index and d is the thickness of the film

$d = \frac{\lambda}{4n}$

(b) $d = \frac{\lambda}{4n} \quad \frac{4.8 \times 10^{-7}}{4 \times 1.25} = \underline{\underline{9.6 \times 10^{-8}}}$ m

5 $d = \frac{\lambda}{4n} = \frac{6.7 \times 10^{-7}}{4 \times 1.3} = \underline{\underline{1.3 \times 10^{-7}}}$ m

6 $d = \frac{\lambda}{4n} \quad \frac{6.2 \times 10^{-7}}{4 \times 1.3} = \underline{\underline{1.2 \times 10^{-7}}}$ m

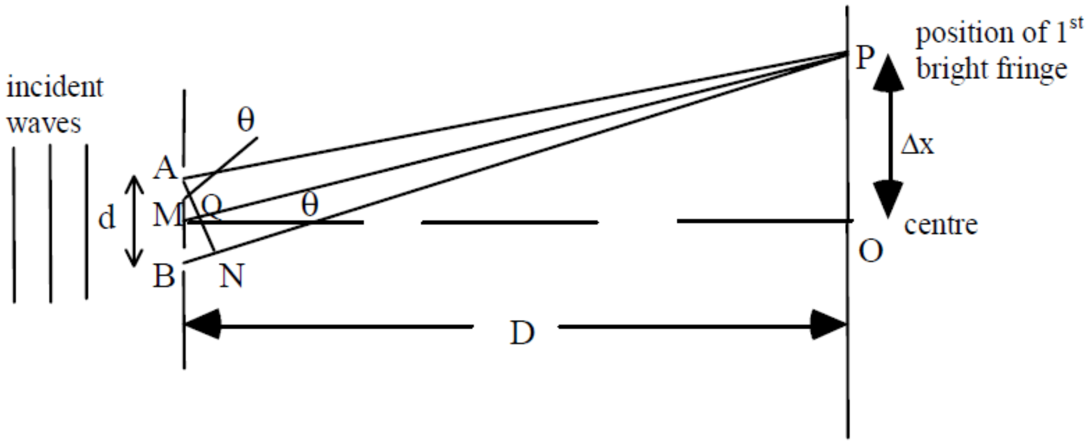
TUTORIAL 4.0

Interference – division of wavelength

1. (a) When light is incident on two small slits, the wavefront is divided and each slit acts as a secondary source. Interference takes place between the two secondary sources.
   (b) With an extended source each part of the wave would be incident on the slit at a different angle which could produce overlapping fringes and the interference pattern would be lost. A point source (or a line source parallel to the slits) must be used.
   (c) With division of amplitude the beam is split at a point with partial reflection and transmission.

2 (a)

The diagram below shows light from a single source of monochromatic light incident on a double slit. The light diffracts at each slit and the overlapping diffraction patterns produce interference.



A bright fringe is observed at P. Angle PMO is θ.
N is a point on BP such that NP = AP. Since P is the first bright fringe BN = λ
For **small** values of θ AN cuts MP at almost 90^0 giving angle MAQ = θ and hence angle BAN = θ.
Again providing θ is **very small**, sin θ = tan θ = θ in radians

From triangle BAN: $\theta = \frac{\lambda}{d}$ also from triangle PMO: $\theta = \frac{\Delta x}{D}$

Thus $\frac{\Delta x}{D} = \frac{\lambda}{d}$ or $\Delta x = \frac{\lambda D}{d}$

Giving the fringe separation between adjacent fringes Δx

$$\boxed{\Delta x = \frac{\lambda D}{d}}$$

Note This formula only applies if x<<D, which gives θ small. This is likely to be true for light waves but not for microwaves.

(b) $x << D$ giving θ a small angle and $\sin\theta = \theta = \tan\theta$

3. (a) $\Delta x = \frac{\lambda D}{d}$ giving $\lambda = 6.0 \times 10^{-7}$ m

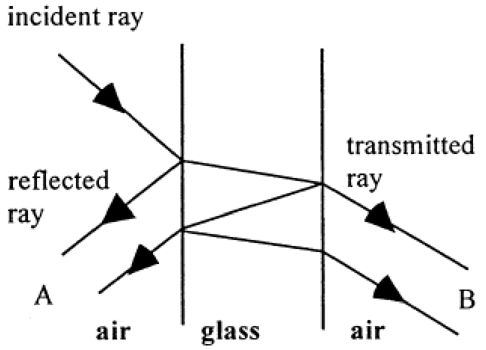
(b) % uncertainty in: Δx is 1.1%; d is 4.2%; D is 2.6%
The 1.1% can be neglected since less than one third of 4.2%
Total uncertainty = $\sqrt{4.2^2 + 2.6^2} = 4.9\%$
Uncertainty in the wavelength = 0.3×10^{-7} m
$\lambda = (6.0 \pm 0.3) \times 10^{-7}$ m

4. (a) Increase the slide to screen distance D.
(b) Fringes are further apart.
(c) The fringes are further apart.
(d) A travelling microscope
(e) Measure the distance across a number of fringes, for example ten, then calculate the fringe spacing.

TUTORIAL 4.1

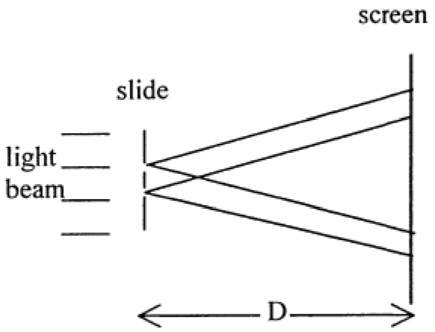
Interference – division of wavefront

1 An example of **division of amplitude** involves splitting a single beam into two beams by producing a reflected beam and a transmitted beam at thin parallel sided film. The reflected rays at A will give interference. Similarly the transmitted rays at B will give interference. In both cases the rays can be brought to a focus with the eye.



An extended source of light can be used. Any transparent boundary between two media of different refractive index can produce division of amplitude.

An example of **division of wavefront** involves a single source incident on a double slit producing two secondary sources. The two secondary beams will give interference fringes on a screen, see sketch below which is not to scale. The distance between the slits must be very small and the distance D is a few metres.



The source of light must be a point or line source. The slits size must be the order of the wavelength to act as secondary sources.

2 (a) The two narrow slits act as coherent sources by division of wavefront. When light from each of the sources meet in phase areas of constructive interference are produced, giving a bright fringe. When the light from the two sources are completely out of phase there is destructive interference and almost darkness.

(b) White fringes have coloured edges because white light is composed of the colours of the spectrum. The position of the nth interference fringe is given by: $x_n = \frac{n\lambda D}{d}$

Notice that the position of a fringe is dependent on the wavelength, λ; as λ increases x_n increases. Thus red will be deviated most and violet least. These two colours will appear at the edges of the white fringes.

3 (a) $\Delta x = \frac{\lambda D}{d} \qquad \Delta x = \frac{695 \times 10^{-9} \times 0.92}{2.0 \times 10^{-4}}$

$\Delta x = 3.2 \times 10^{-3}\ \text{m} = \underline{\underline{3.2}}\ \text{mm}$

(b) The new double slit is half the size. From the relationship in (a) above the fringe separation and the slit separation are inversely proportional. This means that decreasing d will increase x. The pattern will spread out and the fringes will be further apart. In this case the spacing of the fringes will double to 6.4 mm.

4 $\Delta x = \frac{\lambda D}{d}$ thus $\lambda = \frac{\Delta x d}{D}$

$$\lambda = \frac{8 \times 10^{-3} \times 5.0 \times 10^{-4}}{7.2}$$

$$= 5.56 \times 10^{-7} = \underline{\underline{556}} \text{ nm}$$

5 (a) Two coherent sources are produced by the double slit. Interference takes place between these two sources and red and black lines called fringes will be seen on the screen. With a red filter only red fringes are seen because all of the other wavelengths, except red, are absorbed by the filter.

A bright red line is formed by constructive interference. The path difference is nλ, the waves arrive in phase, there is a larger amplitude and more energy.

A dark line is formed by destructive interference. The path difference is (n + ½)λ, the waves are completely out of phase, the amplitude is zero and there is no energy.

(b) The blue fringes will be closer together because the fringe separation Δx is proportional to the wavelength λ and $\lambda_{blue} < \lambda_{red}$.

(c) White fringes have coloured edges because white light is composed of the colours of the spectrum. The position of the nth interference fringe is given by: $x_n = \frac{n\lambda D}{d}$

Notice that this is dependent on λ the wavelength. Thus red will be deviated most and violet least. These two colours will appear at the edges of the white fringes.

(d) $\lambda = \frac{\Delta x d}{D}$

$$= \frac{5 \times 10^{-3} \times 0.25 \times 10^{-3}}{2.0}$$

$$= 6.25 \times 10^{-7} = \underline{\underline{625}} \text{ nm}$$

6 (a) (i) $\Delta x = \frac{\lambda D}{d}$

Δx is proportional to D. Thus making D smaller will also reduce Δx.

(ii) Δx is inversely proportional to d. Thus decreasing d will increase Δx, making the fringes further apart.

(b) (i) Covering one of the slits will cause the interference patterns to disappear. Two sources are needed to produce interference in division of wavefront.

(ii) $\Delta x \; \alpha \; \lambda$

Thus a longer wavelength will produce a larger Δx and the fringes will be further apart.

(iii) White light fringes will be seen. Depending on the values of λ, D and d the fringes may have red and violet coloured edges, see answer to 5 (c).

(c)

$$\lambda = \frac{\Delta x d}{D}$$

$$= \frac{10 \times 10^{-3} \times 0.5 \times 10^{-3}}{8.0}$$

$$= 6.25 \times 10^{-7} = \underline{\underline{625}}\ \text{nm}$$

(d) The fringe separation would have the greatest percentage uncertainty because this is likely to be measured using a metre stick giving (10 ± 1) mm which is 10%. The slit separation can be measured to a higher accuracy with a travelling microscope. The distance to the screen is much larger than the fringe separation so the percentage uncertainty for this will be much less, namely 8 m ± 1 mm which is 0.01%

7 (a) (i)

$$\lambda = \frac{\Delta x d}{D} = \frac{7 \times 10^{-3} \times 0.2 \times 10^{-3}}{2.4}$$

$$= 5.8 \times 10^{-7} = \underline{\underline{580}}\ \text{nm}$$

(ii)

$$\text{uncertainty in } \Delta x = \frac{1}{7} \times 100 = 14\ \%$$

$$\text{uncertainty in } d = \frac{0.01}{0.20} \times 100 = 5\ \%$$

$$\text{uncertainty in } D = \frac{0.10}{2.4} \times 100 = 4\ \%$$

The uncertainty in D can be neglected, since it is less than $\frac{1}{3}$ of 14 %.

$$\text{Total \% uncertainty} = \sqrt{14^2 + 5^2} = 14.9\ \%$$

Thus uncertainty in the wavelength is $\underline{\underline{86}}$ nm and $\lambda = (\underline{\underline{5.8}} \pm \underline{\underline{0.9}}) \times 10^{-7}$ m

(b) (i) Place the double slit slide on the stage of a travelling microscope. Focus the cross hairs in the objective of the microscope on the edge of one of the slits ruled on the slide. Sometimes illumination can be a problem, so try putting a low voltage bulb beneath the microscope stage below the position of the slide. The edge of the slit rulings may now be visible.
Read this position on the vernier scale. Now rack the microscope along the frame until the crosshairs are at the point which gives the slit separation. Read the new position on the vernier scale. To avoid backlash in the mechanism, do not rack the microscope back and forth; i.e. only move it in one direction when taking both readings.

(ii) This could be improved by counting a number of fringes and dividing by the number rather than simply measuring the separation of adjacent fringes.

(c) (i) Reducing d increases the fringe separation Δx because $\Delta x \ \alpha\ \frac{1}{d}$.

(ii) Blue light has a shorter wavelength than yellow light. The blue fringes will therefore be closer together than the yellow fringes because $\Delta x \ \alpha\ \lambda$.

(iii) Covering one of the slits will cause the interference patterns to disappear. Two coherent sources are needed to produce interference.

TUTORIAL 5.0

Polarisation

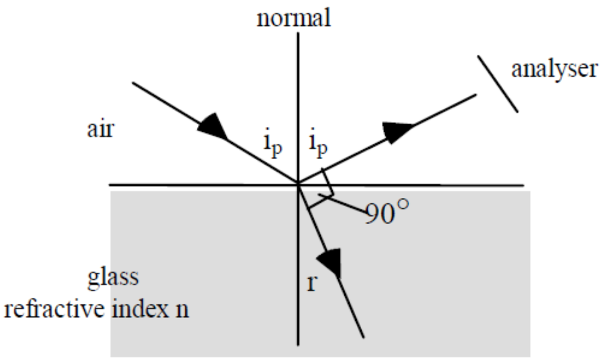
1. (a) With linearly polarised light the oscillations of the electric field strength vector are restricted to one plane. With unpolarised light the electric filed strength vector oscillates in all directions perpendicular to the direction of wave propagation.

   (b) A polaroid filter will only transmit vibrations of the electric field vector in one plane.

   (c) A polariser and analyser are placed at right angle to each other. They are both placed perpendicular to the direction of transmission. The polariser will only transmit vibrations of the electric field vector in one plane, the analyser will absorb these vibrations since they are all perpendicular to its axis of tranmission.

2. (a) The medium is an electric insulator.

   (b)



Consider a beam of unpolarised light incident on a sheet of smooth glass. This beam is partially reflected and partially refracted. The angle of incidence is varied and the reflected ray viewed through an analyser, as shown above. It is observed that at a certain angle of incidence i_p the reflected ray is plane polarised. No light emerges from the analyser at this angle.

The **polarising angle** i_p or **Brewster's angle** is the angle of incidence which causes the reflected light to be linearly polarised.

This effect was first noted by an experimenter called Malus in the early part of the nineteenth century. Later Brewster discovered that **at** the polarising angle i_p the refracted and reflected rays **are separated by 90°.**
Consider the diagram above, which has this 90° angle marked:

$$n = \frac{\sin i_p}{\sin r}$$

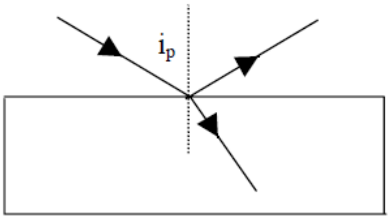
$$\text{but} \quad r = (90 - i_p) \quad \text{thus} \quad \sin r = \sin(90 - i_p) = \cos i_p$$

$$\text{Thus} \quad n = \frac{\sin i_p}{\cos i_p} = \tan i_p$$

$$\boxed{n = \tan i_p}$$

(c) Brewster's angle

3. (a)



(b) $n = \tan i_p$ $i_p = 56^o$

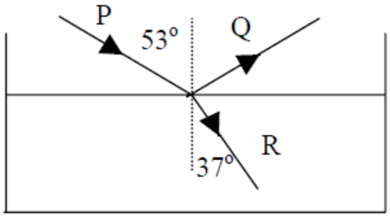
4. When light is reflected from a horizontal surface, such as water, the light will be polarised. The polariod in the sunglasses acts as an analyser and cuts out a large part of the reflected light. (Note that the light is only completely polarised at the Brewster's angle.)

5. (a) $n = \tan i_p$ $i_p = 55^o$
(b) angle of refraction = 90 – 55 = 35°

6. $n = 1/\sin\theta_C = 1.54$ $n = \tan i_p$ $i_p = 57^o$

7. When the frequency increases the refractive index increases. Hence the polarising angle will increase slightly.

8. (a) The intensity of the light observed through the polaroid decreases to a minimum, at the polarising angle, then increases again.
(b) $n = \tan i_p$ $i_p = 53^o$
(c) Angle between ray Q and R is 90°



TUTORIAL 5.1

Polarisation

1 (a) $n = \tan i_p$

thus $\tan i_p = 1.52$

$i_p = \underline{\underline{57°}}$

$r = 90 - i_p$

$= \underline{\underline{33°}}$

(b) Hold a polaroid filter, an analyser, at the appropriate angle of 57° and rotate it. At some point in the rotation no light will be transmitted through the analyser.

2 (a) $r = 90 - i_p$

$= \underline{\underline{38°}}$

(b) $$n = \frac{\sin i}{\sin r} = \frac{\sin 52°}{\sin 38°}$$

$= \underline{\underline{1.28}}$

3 (a) $n = \tan i_p$

red - 650 nm $i_p = \tan^{-1} 1.52$

$= 56.7°$

green - 510 nm $i_p = \tan^{-1} 1.53$

$= 56.8°$

violet - 400 nm $i_p = \tan^{-1} 1.54$

$= 57°$

(b) $r = 90° - i_p$

maximum r: $= 90° - 56.7°$

$= 33.3°$

4 (a) $$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{0.028} = \underline{\underline{1.1 \times 10^{10}}}\text{ Hz}$$

(b) The microwave emitter gives out plane polarised waves. The grid in this arrangement acts as an analyser. The metal grid only transmits when the oscillations of the electric field strength vector are perpendicular to the metal rods. Since the initial reading is low, this suggests that the emitted microwaves are plane polarised with their electric vector in the vertical plane.

As the grid is rotated to the horizontal plane, the reading on the receiver will increase because the microwaves will be transmitted. When the grid is rotated further so that it is upside down, to its initial direction, the reading will be a minimum again. There will be a further maximum when the grid is again horizontal.

(c) The venetian blind is acting like the grid in (a) above. Some of the TV waves are being absorbed by the metal slats of the blind.

5 $$n = \frac{1}{\sin c} = \frac{1}{\sin 38°} = 1.62$$

$$i_p = \tan^{-1} 1.62 = \underline{\underline{58}}°$$

6. (a) When light is incident at a boundary between air and an electrical insulator, the polarising angle i_p is the angle of incidence in air which causes the reflected light to be linearly polarised.

(b) Brewster's angle

(c) Assumption: at the polarising angle the refracted and reflected rays are separated by 90°

$$n = \frac{\sin i_p}{\sin r} \qquad \text{and } r = 90 - i_p$$

$$n = \frac{\sin i_p}{\cos i_p} \qquad \text{since } \sin(90 - i_p) = \cos i_p$$

thus $n = \tan i_p$

(d) (i) $r = 90 - i_p = \underline{\underline{34}}°$

(ii) $n = \dfrac{\sin i}{\sin r} = \underline{\underline{1.48}}$

7 (a) (i)

$$n = \tan i_p$$

$$i_p = \tan^{-1} 1.33$$

$$= \underline{\underline{53}}°$$

(ii) angle of refraction = $90° - 53° = \underline{\underline{37}}°$

(b) (i) The path of the light is reversed for the same polarising condition:

$$i_p = \underline{\underline{37}}°$$

Alternatively $${}_{\text{water}}n_{\text{air}} = \frac{1}{{}_{\text{air}}n_{\text{water}}} = \frac{1}{1.33}$$

$$i_p = \tan^{-1} \frac{1}{1.33} = 37°$$

(ii) The refracted ray in air will be $\underline{\underline{53}}°$