Wallace Hall Academy Physics Department

Advanced Higher Physics

Electromagnetism

Solutions

Tutorial 1.0

Coulomb's inverse square law and electric field strength

1. $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$

   where F - force between the two charges (N), Q_1 and Q_2 – charges (C)
   ε_0 – permittivity of free space (F m^{-1}), r – separation of charges (m)

2. $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ $\quad F = 9 \times 10^9 \times \dfrac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(1.5 \times 10^{-9})^2}$

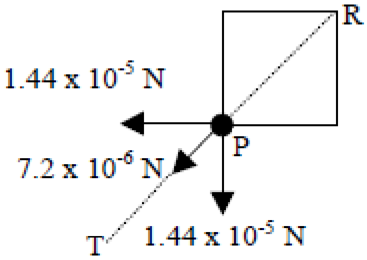
   $F = 1.0 \times 10^{-10}$ N repulsion [the direction must be stated]

3. $14 = 9 \times 10^9 \times \dfrac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{r^2}$ giving $r = 4.1 \times 10^{-15}$ m

4. (a) Magnitude of F due to charge Q $= 9 \times 10^9 \times \dfrac{4 \times 10^{-9} \times 4 \times 10^{-9}}{0.1^2}$

   $= 1.44 \times 10^{-5}$ N

   Magnitude of F due to charge S $= 1.44 \times 10^{-5}$ N (same separation)
   Magnitude of F due to charge R $= 7.2 \times 10^{-6}$ N (r = 0.1414 m)
   Total force on charge is determined by vector addition.



   Combining the two 1.44×10^{-5} N forces gives a force of 2.04×10^{-5} N in the same direction as the 7.2×10^{-6} N force.

   Hence total force on P is $= (2.04 + 0.72) \times 10^{-5}$ N
   $F = 2.8 \times 10^{-5}$ N
   in the direction RPT, shown on the diagram.

   (b) zero. The two 4.0 nC charges at opposite ends of a diagonal will exert an equal and opposite force on the –1.0 nC charge at the centre, hence the resultant force will be zero.

5. $F = 9 \times 10^9 \times \frac{20 \times 10^{-9} \times 20 \times 10^{-9}}{0.8^2} + 9 \times 10^9 \times \frac{20 \times 10^{-9} \times 20 \times 10^{-9}}{1.6^2}$

$F = 7.0 \times 10^{-6}$ N to the right

6. The electric field strength is the electrostatic force on one coulomb of charge placed at that point.

7. (a) $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ (b) $E = \frac{V}{d}$

8. See Student Material page 5.

9. $E = \frac{Q}{4\pi\varepsilon_0 r^2} = 9 \times 10^9 \times \frac{2 \times 1.6 \times 10^{-19}}{(1 \times 10^{-10})^2}$

$= 2.88 \times 10^{11}$ V m^{-1} (or N C^{-1}) away from the nucleus

10. $E = \frac{Q}{4\pi\varepsilon_0 r^2}$; $72\ 000 = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{r^2}$ $r = 0.50$ m

11. (a) Towards the bottom plate, perpendicular to the plates.

(b) (i) $E = \frac{V}{d} = \frac{4.0 \times 10^3}{0.02}$ $= 2.0 \times 10^{-5}$ V m^{-1}

(ii) The same $E = 2.0 \times 10^{-5}$ V m^{-1}, the field is uniform between the plates.

Tutorial 1.1

Coulomb's Inverse Square Law

Note: the following examples use $\frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

1 (a) Use Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$$

$$= 9.0 \times 10^9 \times \frac{2 \times 10^{-8} \times 4 \times 10^{-8}}{(2 \times 10^{-2})^2}$$

$$= \underline{\underline{0.018}} \text{ N}$$

This is a force of attraction since the charges have opposite sign.

(b) $F = 1.0 \times 10^{-4}$ N

$$r^2 = \frac{1}{4\pi\varepsilon_o} \times \frac{Q_1Q_2}{F}$$

$$= 9.0 \times 10^9 \times \frac{(2 \times 10^{-8} \times 4 \times 10^{-8})}{1.0 \times 10^{-4}}$$

thus $r = \underline{\underline{0.27}}$ m

2 $F_g = \frac{GM_1M_2}{r^2}$ $F_e = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$

We could work out the forces separately. However it is easier to simply take the ratio $\frac{F_e}{F_g}$. Then the r^2 will cancel.

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_o} \times Q_1 Q_2}{G M_1 M_2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6.67 \times 10^{-11} \times 1.673 \times 10^{-27} \times 9.11 \times 10^{-31}}$$

$$= \underline{\underline{2.3 \times 10^{39}}}$$

Thus the electrostatic force is almost 10^{40} times bigger than the gravitational force between sub-atomic particles. We can therefore safely neglect gravitational effects for such particles.

3 Here $F_e = F_g$

thus $\frac{1}{4\pi\varepsilon_o}\frac{Q\,Q}{r^2} = \frac{GMm}{r^2}$ r^2 cancels

$$9 \times 10^9 \times Q^2 = 6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.3 \times 10^{22}$$

$$Q^2 = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.3 \times 10^{22}}{9 \times 10^9}$$

and $Q = \underline{\underline{5.7 \times 10^{13}}}$ C

This is a huge charge and as you will see later it is not possible to create a positive charge in isolation. Such a possibility for the Earth could not arise.

4

$$F_2 = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$$

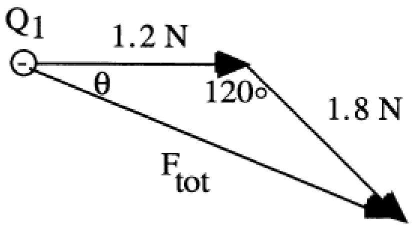
$$= 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 3 \times 10^{-6}}{(0.15)^2}$$

$= 1.2$ N in a direction from Q_1 towards Q_2

$$F_3 = 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 2 \times 10^{-6}}{(0.10)^2}$$

$= 1.8$ N in a direction away from Q_1, along the line Q_3Q_1

The resultant force is the vector sum of these two forces.



Use the cosine rule:

$$F^2_{tot} = 1.2^2 + 1.8^2 - 2 \times 1.2 \times 1.8 \cos 120^\circ$$

$$= 6.84$$

thus $F_{tot} = \sqrt{6.84} = \underline{\underline{2.6}}$ N

To find the direction, θ, use the sine rule:

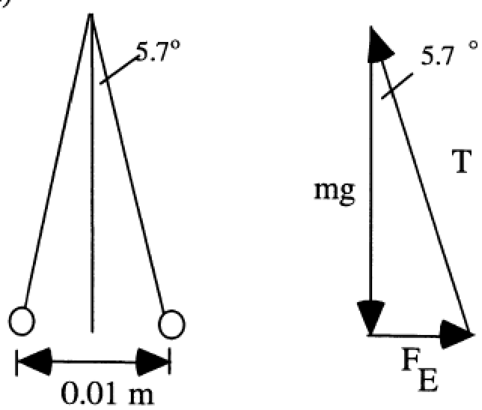
$$\frac{\sin\theta}{1.8} = \frac{\sin 120^\circ}{2.6}$$

$$\theta = \underline{\underline{37}}^\circ$$

The resultant force is 2.6 N at 37° as shown in the sketch opposite.

This problem could have been done by scale drawing.

5 (a)



$$F_E = mg \tan 5.7^\circ$$

$$= 0.10 \times 10^{-3} \times 9.8 \times \tan 5.7$$

$$= 9.78 \times 10^{-5}$$

$= \underline{\underline{9.8 \times 10^{-5}}}$ N (2 sig. figs.)

Alternatively use the components of the tension **T**.

(b) $F_E = \frac{1}{4\pi\varepsilon_o}\frac{QQ}{r^2}$ thus $Q^2 = \frac{9.8 \times 10^{-5} \times (0.01)^2}{9 \times 10^9}$

$Q = \underline{\underline{1.04 \times 10^{-9}}}$ C

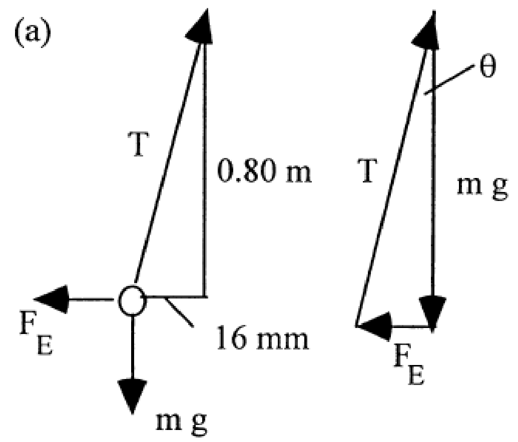
(c) $Q = It$ thus $t = \frac{Q}{I}$

$$t = \frac{1.04 \times 10^{-9}}{1 \times 10^{-11}} = \underline{\underline{104}} \text{ s}$$

(d) Charge one of the spheres by touching it to the dome of a Van de Graaff generator. Now touch the uncharged sphere to the charged sphere. If both spheres are the same size, the charge will be shared equally.

6

(a)

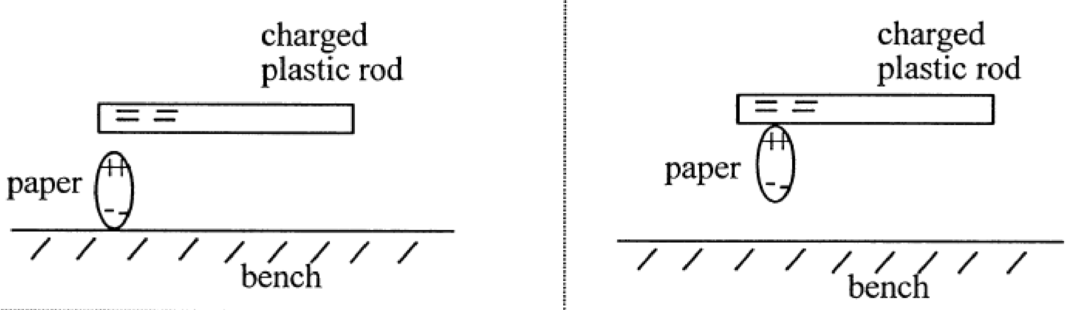


(b)

$$\tan\theta = \frac{0.016}{0.80} = 0.02$$

$$F_E = mg\tan\theta$$
$$= 0.50 \times 10^{-3} \times 9.8 \times 0.02$$
$$= \underline{\underline{9.8 \times 10^{-5}}}\ \text{N}$$

7



The charged plastic rod causes the polar molecules in the paper to line up in the way shown. Note that the paper is overall neutral. The paper is attracted because the positive charge is closer to the negative rod.

Tutorial 2.0

Electric fields and electrostatic potential

1. (a) zero

   (b) $E = \dfrac{Q}{4\pi\varepsilon_0 r^2} = 9 \times 10^9 \times \dfrac{30 \times 10^{-6}}{0.04^2} = 1.7 \times 10^8\ \text{V m}^{-1}$ away from the sphere.

   (c) $E = 2.7 \times 10^5\ \text{V m}^{-1}$ [as above with r = 1 m] away from the sphere.

2. See Student Material page 6. Notice that the object charged by induction, the electroscope, is not touched by the charging object, the negatively charged rod.

3. The electrostatic potential at a point is the work done to bring one coulomb of charge from infinity to that point.

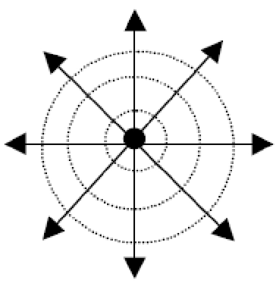
4. $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

5. $V = \dfrac{Q}{4\pi\varepsilon_0 r} = 9 \times 10^9 \times \dfrac{4 \times 10^{-9}}{3} = 12\ \text{V}$

6. $V_A = 9 \times 10^9 \times \dfrac{6 \times 10^{-9}}{2} = 27\ \text{V}$ and $V_B = 9 \times 10^9 \times \dfrac{-6 \times 10^{-9}}{5} = -10.8\ \text{V}$

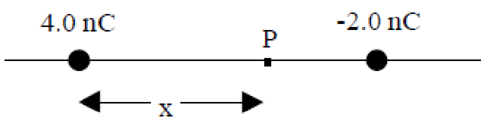
   Potential difference $V_{AB} = 16.2\ \text{V}$ where A is negative compared to B

7. A surface on which the potential is the same at all points. No work would be done to move a charge between two points on the surface.

8. 

   The field lines are full lines and the lines of equipotential are broken lines.

   Notice that the field lines and lines of potential are perpendicular to each other.

9. 

   For zero potential at point P: V(due to 4.0 nC) + V(due to -2.0 nC) = 0

   $9 \times 10^9 \times \dfrac{4 \times 10^{-9}}{x} + 9 \times 10^9 \times \dfrac{-2 \times 10^{-9}}{(0.12 - x)} = 0$

   $4(0.12 - x) - 2x = 0$ and $0.48 - 4x - 2x = 0$ giving $x = 0.08$ m

10. Electrostatic force, electric field strength.

11. (a) $V_X = 9 \times 10^9 \times \frac{2.5 \times 10^{-9}}{0.1} + 9 \times 10^9 \times \frac{2.5 \times 10^{-9}}{0.412} = 280\ V$

$V_Y = 2 \times (9 \times 10^9 \times \frac{2.5 \times 10^{-9}}{0.224}) = 201\ V$

(b) $V_{XY} = 79\ V$

12. Energy = VQ $\quad 10^{-14} = V \times 1.6 \times 10^{-19}$ giving V = 62.5 kV

Tutorial 2.1

Electric Field Strength

1 $$E = \frac{F}{Q} = \frac{0.02}{4.0 \times 10^{-6}}$$
$$= \underline{\underline{5000}}\ \text{N C}^{-1}$$

2 (a) $$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

$$1.0 = 9 \times 10^9 \times \frac{Q}{(1.0)^2}$$

thus $$Q = \frac{1}{9 \times 10^9} = \underline{\underline{1.1 \times 10}}^{-10}\ \text{C}$$

(b) $E \propto \frac{1}{d^2}$ the distance has been doubled thus field strength quarters.

E at 2.0 m will be $\underline{\underline{0.25}}$ N C^{-1}

3 (a) $$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

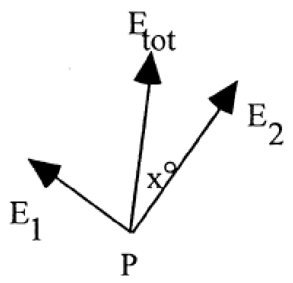
α-particle has 2 protons $Q = 2 \times 1.6 \times 10^{-19}$ C

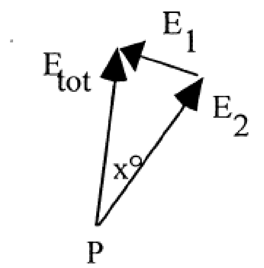
thus $$E = 9 \times 10^9 \times \frac{3.2 \times 10^{-19}}{(5 \times 10^{-3})^2}$$
$$= \underline{\underline{1.2 \times 10}}^{-4}\ \text{N C}^{-1}$$

(b) for one proton the charge is halved compared to part (a) and $E \propto Q$

thus E will also be halved: $E = \underline{\underline{6.0 \times 10}}^{-5}$ N C^{-1}

4 (a)





The angle at P is a right angle.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q_1}{r^2} = 9 \times 10^9 \times \frac{18.8 \times 10^{-9}}{(0.12)^2}$$
$$= 1.2 \times 10^4\ \text{N C}^{-1}\ \text{(2 sig figs)}$$

in direction shown, from 18.8 nC charge

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{Q_2}{r^2} = 9 \times 10^9 \times \frac{10 \times 10^{-9}}{(0.05)^2}$$
$$= 3.6 \times 10^4\ \text{N C}^{-1}$$

in direction shown, from 10 nC charge

$$E_{tot} = \sqrt{E_1^2 + E_2^2}\ \text{(magnitude)}$$
$$= \underline{\underline{3.8 \times 10}}^4\ \text{N C}^{-1}$$

(b) From the sketch above:

$$\tan x^\circ = \frac{E_1}{E_2} = \frac{1.2 \times 10^4}{3.6 \times 10^4}$$
$$x = \underline{\underline{18}}^\circ$$

5 (a) For a stationary charged sphere:

the downward force of gravity = upward electric force

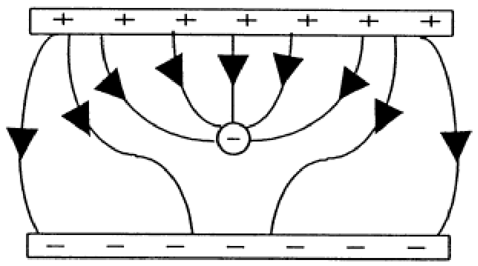
thus $mg = EQ$

$$E = \frac{mg}{Q}$$

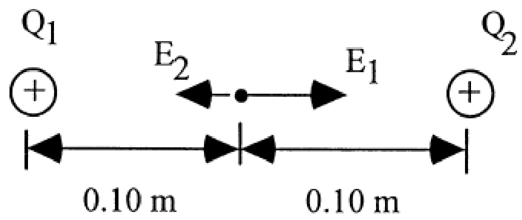
$$E = \frac{2.0 \times 10^{-5} \times 9.8}{5.0 \times 10^{-9}}$$

$$= \underline{\underline{3.9 \times 10^{4}}}\ \text{N C}^{-1}$$

(b)



6 (a)



E_1 will be double the size of E_2 because Q_1 is double the size of Q_2.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q_1}{r_1^2} = 9 \times 10^9 \times \frac{8.0 \times 10^{-9}}{(0.10)^2} = 7.2 \times 10^3\ \text{N C}^{-1}$$

thus $E_2 = 3.6 \times 10^3\ \text{N C}^{-1}$ Directions of E_1 and E_2 are shown in sketch

$E_{tot} = E_1 - E_2 = \underline{\underline{3.6 \times 10^3}}\ \text{N C}^{-1}$ in the direction from Q_1 to Q_2.

(b) When $E_{tot} = 0\ \text{N C}^{-1}$ $E_1 = -E_2$ and in magnitude $E_1 = E_2$

$$\frac{1}{4\pi\varepsilon_o}\frac{Q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_o}\frac{Q_2}{r_2^2} \quad \text{and} \quad r_2 = (0.20 - r_1)$$

$$\frac{Q_1}{r_1^2} = \frac{Q_2}{(0.20 - r_1)^2}$$

thus $\dfrac{Q_1}{Q_2} = \dfrac{r_1^2}{(0.20 - r_1)^2}$ but $Q_1 = 2Q_2$

thus $2 = \dfrac{r_1^2}{(0.20 - r_1)^2}$ take square root of each side

$$\sqrt{2} = \frac{r_1}{(0.20 - r_1)}$$

thus $r_1 = \sqrt{2} \times (0.20 - r_1)$ and $0.20 \times \sqrt{2} = (1 + \sqrt{2})r_1$

$$r_1 = \frac{0.20 \times \sqrt{2}}{(1 + \sqrt{2})} = \underline{\underline{0.12}}\ \text{m}$$

(c) (i) Use Coulomb's Law: $F = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r^2}$

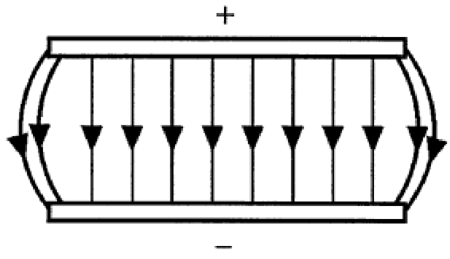
$$= 9 \times 10^9 \times \frac{8.0 \times 10^{-9} \times 4.0 \times 10^{-9}}{(0.2)^2}$$

$$= \underline{\underline{7.2 \times 10^{-6}}}\ \text{N}$$

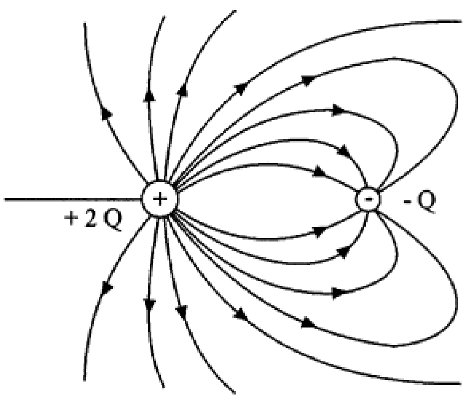
(ii) Using $F = mg$ $F = 5\ 0 \times 10^{-4} \times 9.8$

$$F = \underline{\underline{4.9 \times 10^{-3}}}\ \text{N}$$

7 (a)



(b)



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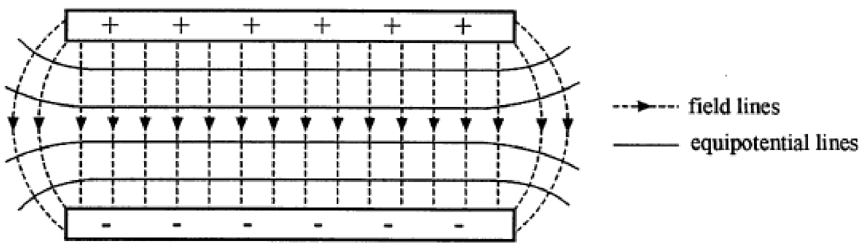


field lines and equipotential lines are always at right angles

Tutorial 3.1

Electrostatic Potential

1 $$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = 9 \times 10^9 \times \frac{3 \times 10^{-9}}{005}$$
$$= \underline{540} \text{ V}$$

2 (a) Potential is a scalar - therefore there is no need to consider any directions.
Note that all the charges are equidistant from C at 0.10 m.
Potential due to a +1.0 nC charge at C:

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} = 9 \times 10^9 \times \frac{1.0 \times 10^{-9}}{0.10} = 90 \text{ V}$$

Potential due to a negative charge is negative.

$$V_C = (2 \times 90) + (-2 \times 90) + (6 \times 90) + (3 \times 90)$$
$$= 180 - 180 + 540 + 270$$
$$= \underline{810} \text{ V}$$

(b) Distance from +2.0 nC and +6.0 nC to D = 0.158 m (by Pythagoras)
Distance from -2.0 nC and +3.0 nC to D = 0.0707 m (by Pythagoras).

thus $$V \text{ due to } +6.0 \text{ nC at D} = 9 \times 10^9 \times \frac{6.0 \times 10^{-9}}{0.158} = 341.8 \text{ V}$$

$$V \text{ due to } +2.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.158} = 113.9 \text{ V}$$

$$V \text{ due to } -2.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.0707} = -254.6 \text{ V}$$

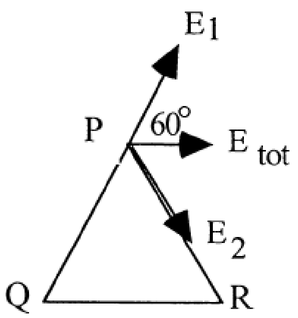
$$V \text{ due to } +3.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.0707} = -381.9 \text{ V}$$

$$V_{tot} = 583 \text{ V at D}$$

thus potential difference between C and D = 810 - 583 = $\underline{+227}$ V

3 $$\text{Energy} = QV = Q \times \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$
$$= 1.6 \times 10^{-19} \times 9 \times 10^9 \times \frac{1.6 \times 10^{-19}}{5 \times 10^{-11}}$$
$$= \underline{4.6 \times 10^{-18}} \text{ J}$$

4 (a)

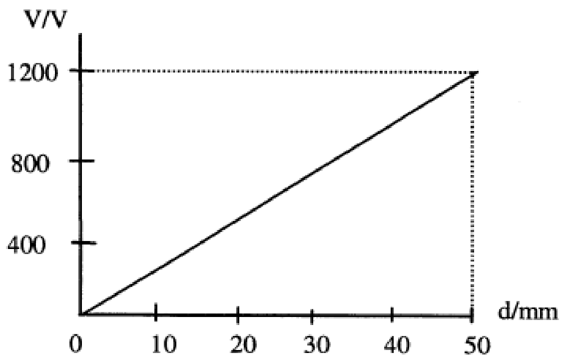


$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} = 9 \times 10^9 \times \frac{1 \times 10^{-8}}{(2.0 \times 10^{-2})^2}$$
$= 2.25 \times 10^5$ N C^{-1} in direction away from Q
$E_2 = 2.25 \times 10^5$ N C^{-1} from P towards R

$$E_{tot} = E_1 \cos 60^\circ + E_2 \cos 60^\circ$$
$= \underline{2.25 \times 10^5}$ N C^{-1} in the direction shown

(b) The potential at P will be zero because charges at Q and R are equal in size and opposite in sign and both points are equidistant from P.

5 (a)



(b) $E = \frac{V}{d} = \frac{1200}{5 \times 10^{-2}}$

$= \underline{2.4 \times 10^4}\ \text{V m}^{-1}$

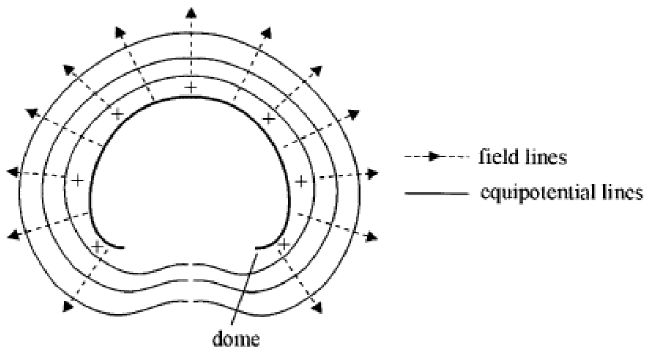
Direction of field is toward the lower plate

(c) Take the gradient of the graph

$E = -\frac{dV}{dx}$

6 (a) Equipotential surfaces have the same potential at all points. Note that moving a charge between two points on an equipotential surface needs no work.

(b) In the sketch below the solid lines show the electric field and the dotted lines show the equipotential surface lines.



7

$$E = \frac{V}{d} = \frac{1500}{0.02}$$

$$= \underline{7.5 \times 10^4}\ \text{V m}^{-1}$$

8 (a)

$$E = \frac{V}{d} = \frac{2000}{0.15}$$

$$= \underline{1.33 \times 10^4}\ \text{V m}^{-1}$$

(b) (i) Energy change: electrical potential energy into kinetic energy

(ii) work done = Q V

$= 1.6 \times 10^{-19} \times 2000$

$= \underline{3.2 \times 10^{-16}}\ \text{J}$

(iii)

$$QV = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2QV}{m}}$$

$$= \underline{2.7 \times 10^7}\ \text{m s}^{-1}$$

9 (a) The proton will have a uniform acceleration of 1.27×10^{-12} m s^{-2} towards the negative plate, assuming air resistance is negligible. Any downward force due to gravity has been ignored.

(b) (i) The work done will be the same as in the previous question because the charge on the proton is the same as the charge on the electron.

(ii) Using $QV = \frac{1}{2} m v^2$

$$v = \sqrt{\frac{2QV}{m}}$$

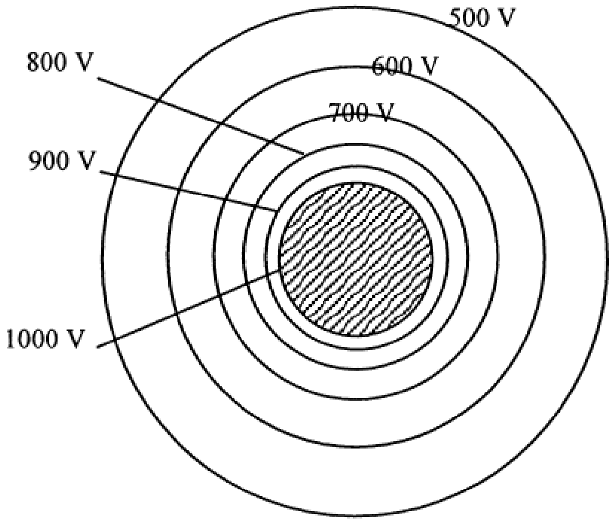
$$= \underline{6.2 \times 10^5 \text{ m s}^{-1}}$$

10 (a) $V = \frac{1}{4\pi\varepsilon_o} \frac{Q}{r}$ thus $V \alpha \frac{1}{r}$ or $V_1 r_1 = V_2 r_2$

thus $r_2 = \frac{V_1 r_1}{V_2} = \frac{0.05 \times 1000}{900} = 0.056$ m

$r_3 = \frac{0.05 \times 1000}{800} = 0.063$ m

similarly, $r_4 = 0.071$ m; $r_5 = 0.083$ m; $r_6 = 0.10$ m



Notice that equipotential lines which are separated by the same amount of p.d., in this case 100 V, become further apart as the radius increases.

(b) $V = \frac{1}{4\pi\varepsilon_o} \frac{Q}{r}$

thus $1000 = 9 \times 10^9 \times \frac{Q}{0.05}$

$Q = \frac{1000 \times 0.05}{9 \times 10^9} = \underline{5.6 \times 10^{-9} \text{ C}}$

11 (a) $v = \sqrt{\frac{2 e V}{m}} = \underline{5.9 \times 10^8 \text{ m s}^{-1}}$

(b) This calculated velocity is greater than the speed of light which is not physically possible. A relativistic calculation is needed, an example is shown for interest at the end of the Mechanics unit solutions.

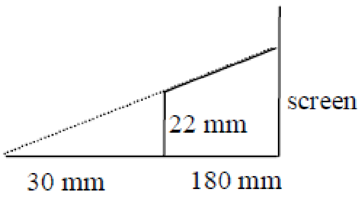
Tutorial 4.0

Charges in motion

1. (a) Work = p.d. x charge = $1000 \times 6.0 \times 10^{-6}$ = 6.0 mJ
   (b) Electrostatic potential energy is transformed into kinetic energy as the charge is accelerated towards the top plate.

2. (a) V = Ed $3.0 \times 10^4 = 5.0 \times 10^5 \times d$ giving d = 0.060 m or 60 mm
   (b) The electric field strength will double.
   (c) change in E_k = change in electrical energy
   $\frac{1}{2}mv^2 - 0 = VQ$ where V is the p.d.
   $v = \sqrt{\frac{2Ve}{m}}$ where e is the charge, and m the mass, of an electron.

3. (a) In the horizontal direction: velocity of electron entering the plates
   $\frac{1}{2}mv^2 = 2.88 \times 10^{-16}$ thus $v = 2.51 \times 10^7 \text{ m s}^{-1}$.
   (b) In the horizontal direction:
   Time taken to travel pass plates at 0.15 m long = 5.98×10^{-9} s
   In the vertical direction: time = 5.98×10^{-9} s initial velocity = 0
   Force on electron due to electric field = EQ = $1.4 \times 10^4 \times 1.6 \times 10^{-19}$ N
   Acceleration = $\frac{F}{m} = \frac{1.4 \times 10^4 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} = 2.46 \times 10^{15} \text{ m s}^{-2}$
   Using $s = ut + \frac{1}{2}at^2$ deflection = 0.044 m
   (c) The electron will travel in a straight line. There is no unbalanced force on the electron because there is no electric field outside the plates.

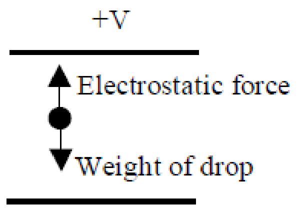
4. (a) $\frac{1}{2}mv^2 = Ve$ $\frac{1}{2} \times 9.11 \times 10^{-31} \times v^2 = 200 \times 1.6 \times 10^{-19}$
   thus $v = 8.38 \times 10^6 \text{ m s}^{-1}$
   (b) In the horizontal direction, time taken = $\frac{\text{length of plates}}{\text{horizontal speed}} = 3.58 \times 10^{-9}$ s
   In the vertical direction, $a = \frac{F}{m} = \frac{1.0 \times 10^3 \times 1.6 \times 10^{-19}}{50 \times 10^{-3} \times 9.11 \times 10^{-31}} = 3.51 \times 10^{15} \text{ m s}^{-2}$
   Initial velocity vertically = 0 hence using $s = ut + \frac{1}{2}at^2$ gives s = 0.022 m
   (c)


   The electron travels in a straight line after leaving the plates.
   By proportion $\frac{\text{deflection}}{22} = \frac{210}{30}$
   Giving the deflection on the screen = 154 mm

5. (a) $1.6 \times 10^{-19} \times 125 \times 10^3 = \frac{1}{2} \times 9.11 \times 10^{-31} \times v^2$ gives $v = 2.09 \times 10^8 \text{ m s}^{-1}$
   (b) This speed is more than 60% of the speed of light. Relativistic effects must be considered when speeds are greater than 10 % of the speed of light.

6. Millikan determined the charge on a number of small charged drops. He noticed that the charges were all multiples of a certain smallest charge, 1.6×10^{-19} C. This suggested that it was not possible to obtain a charge with a fraction of this value.

7. (a)



(The charge on the drop is negative)

(b) weight down = electrostatic force upwards

$$mg = EQ = \frac{VQ}{d}$$

$$0.01 \times 10^{-9} \times 9.8 = \frac{V \times 5 \times 1.6 \times 10^{-19}}{20 \times 10^{-3}}$$

$$V = 2.45 \times 10^{6}\ \text{V}$$

(c) The drop would accelerate upwards, since the electrostatic force is now greater than the weight of the drop. The drop will accelerate in the direction of the resultant force.

8. (a) Excess charge on each drop is calculated using $Q = \frac{mgd}{V}$ and gd = 0.392

Mass of drop/10^{-15} kg	2.6	1.2	1.6	2.3	4.8	5.9	1.8	3.7
p.d. / V	1592	2940	1960	2818	2940	14455	1470	4533
Charge/ 10^{-19} C	6.4	1.6	3.2	3.2	6.4	1.6	4.8	3.2
No. excess electrons	4	1	2	2	4	1	3	2

(b) These are all whole numbers. No fractional charges were found.

9. (a) Change in $E_k = \frac{1}{2} mv^2 - 0$ with zero E_k at the distance of closest approach.

(b) Change in electrostatic $E_p = \frac{qQ}{4\pi\varepsilon_0 r} - 0$ [potential energy = V x charge]

where q and Q are the charges on the alpha particle and oxygen atom.

(c) At closest approach change in E_k = change in E_p

$$\frac{1}{2} mv^2 - 0 = \frac{qQ}{4\pi\varepsilon_0 r} - 0$$

rearranging gives $r = \frac{2qQ}{4\pi\varepsilon_0 mv^2}$

(d) $r = 9 \times 10^9 \times \frac{2 \times (2 \times 1.6 \times 10^{-19}) \times (8 \times 1.6 \times 10^{-19})}{6.7 \times 10^{-27} \times (1.9 \times 10^{6})^2} = 3.0 \times 10^{-13}$ m

10. From above $v^2 = \frac{2qQ}{4\pi\varepsilon_0 mr} = 9 \times 10^9 \times \frac{2 \times (2 \times 1.6 \times 10^{-19}) \times (26 \times 1.6 \times 10^{-19})}{6.7 \times 10^{-27} \times 1.65 \times 10^{-13}}$

and $v = 4.7 \times 10^{6}$ m s^{-1}

Tutorial 4.1

Charges in Motion

1 (a)

$$F = E\,Q$$
$$= 1.2 \times 10^{6} \times 1.6 \times 10^{-19}$$
$$a = \frac{F}{m} = \frac{1.2 \times 10^{6} \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} = 2.11 \times 10^{-17}$$
$$= \underline{2.1 \times 10}^{17}\ \text{m s}^{-2}$$

(b) (i) Using $v = u + a\,t$ $u = 0$ gives $t = \frac{v}{a}$

$$t = \frac{3.0 \times 10^{7}}{2.11 \times 10^{17}} = \underline{1.42 \times 10}^{-10}\ \text{s}$$

(ii) Using $s = ut + \frac{1}{2}\,a\,t^2$

$$= 0 + \frac{1}{2} \times 2.11 \times 10^{17} \times (1.42 \times 10^{-10})^2$$
$$= 2.1 \times 10^{-3}\ \text{m} = \underline{2.1}\ \text{mm}$$

2 (a) The oil drop must be **negatively** charged.

(b) Calculate the size of the electric field: $E = \frac{V}{d} = \frac{2000}{0.02} = 1.0 \times 10^{5}\ \text{N C}^{-1}$

At balance:
$$F_{elect.} = F_{grav.}$$
$$E\,Q = m\,g$$
$$Q = \frac{m\,g}{E} = \frac{4.9 \times 10^{-15} \times 9.8}{1.0 \times 10^{5}}$$
$$= \underline{4.8 \times 10}^{-19}\ \text{C}$$

This is equivalent to an **excess** of three electrons on the oil drop.

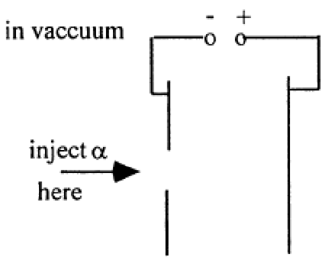
3 (a) Use the principle of conservation of energy:
thus work done on a charge by the electric field = E_k 'lost'

$F \times d = \frac{1}{2}\,m\,v^2$ also: $F = E\,Q$: $E\,Q\,d = \frac{1}{2}\,m\,v^2$

$$\text{thus}\quad E = \frac{\frac{1}{2} m\,v^2}{Q\,d} = \frac{\frac{1}{2} \times 6.7 \times 10^{-27} \times (5 \times 10^{6})^2}{3.2 \times 10^{-19} \times 6.0 \times 10^{-2}}$$
$$E = \underline{4.4 \times 10}^{6}\ \text{N C}^{-1}$$

(Alternatively use: $v^2 = u^2 + 2as$, $F_{un} = ma$ and $E = \frac{F_{un}}{m}$)

(b)



(c) Gamma rays cannot be stopped by an electric field because gamma rays do not have any electric charge. Gamma rays are electromagnetic radiation of very high frequency.

4 (a) $t = \frac{d}{v} = \frac{5.0 \times 10^{-2}}{6.0 \times 10^{6}}$

$= \underline{\underline{8.33 \times 10^{-9}}}$ s (keeping 3 sig figs for parts (b) to (d))

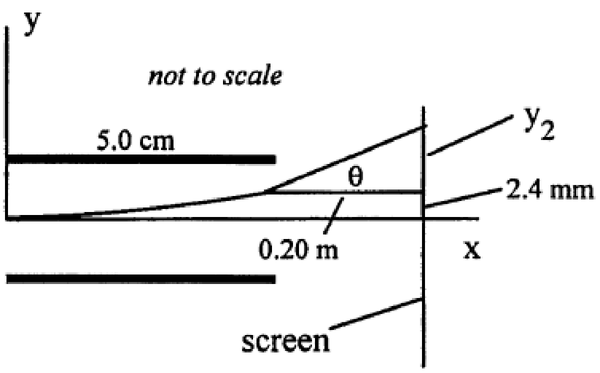
(b) Let the displacement in the vertical plane be y.

$y = u_y t + \frac{1}{2} a t^2$ where $a = \frac{F}{m} = \frac{EQ}{m}$

$u_y = 0$ thus $y = \frac{1}{2} \times \frac{4 \times 10^2 \times 1.6 \times 10^{-19} \times (8.33 \times 10^{-9})^2}{9.11 \times 10^{-31}} = \underline{\underline{2.4 \times 10^{-3}}}$ m

Thus the vertical displacement experienced by an electron is 2.4 mm.

(c)



θ can be worked out from the combination of vertical and horizontal velocities at the plate edge.

(From above $a = \frac{EQ}{m} = 7.025 \times 10^{13}$ m s^{-2})

$v_{vert} = a t = 7.025 \times 10^{13} \times 8.33 \times 10^{-9}$

$= 5.85 \times 10^{5}$ m s^{-1}

$v_{hor} = 6.0 \times 10^{6}$ m s^{-1}

$\tan\theta = \frac{v_{vert}}{v_{hor}} = \frac{5.85 \times 10^5}{6.0 \times 10^6}$

$\theta = \underline{\underline{5.6°}}$

(d) Also $\tan\theta = \frac{y_2}{0.20}$ thus $y_2 = \tan\theta \times 0.20$

$y_2 = 1.96 \times 10^{-2}$ m $= 19.6$ mm

thus total vertical displacement at screen = 2.4 mm + 19.6 mm = 22.0 mm

thus $y_{tot} = \underline{\underline{22 \text{ mm}}}$ (2 sig figs)

5 $QV = \frac{1}{2} m v^2$

giving $v = \sqrt{\frac{2QV}{m}}$

$= \underline{\underline{5.13 \times 10^{8}}}$ m s^{-1}

This speed is greater than the speed of light. Hence this speed is not possible. A relativistic calculation is needed which gives v = 2.7 x 10^8 m s^{-1}. An example is shown, for interest, at the end of the solutions to the Mechanics unit. Such calculations are **not** required for examination purposes.

6 (a) Head on collisions of α-particles with gold nuclei happen very rarely because the nucleus of the gold atom is so very small compared to the overall size of the atom. Most of the atom is empty space.

(b) At closest approach $E_p = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{r}$ q = electronic charge and

Q = charge of gold nucleus

E_k of α–particle $= \frac{1}{2} m v^2$

Change of E_k = change of E_p where r is the closest distance of approach.

$\frac{1}{2} m v^2 - 0 = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{r} - 0$ (initial $E_p = 0$ and final $E_k = 0$)

thus $r = \frac{1}{4\pi\varepsilon_o}\frac{2qQ}{mv^2}$

$$= 9 \times 10^9 \times \frac{2 \times 3.2 \times 10^{-19} \times 79 \times 1.6 \times 10^{-19}}{6.7 \times 10^{-27} \times (2.0 \times 10^7)^2}$$

$$= \underline{\underline{2.7 \times 10^{-14}}}\ \text{m}$$

7 (a) $m = \rho V = 870 \times \frac{4}{3} \pi \times (1.62 \times 10^{-6})^3$

$= \underline{\underline{1.55 \times 10^{-14}}}$ kg

(b) At balance $F_{elect} = F_{grav}$

$E Q = m g$

thus $Q = \frac{m g}{E} = \frac{1.55 \times 10^{-14} \times 9.8}{1.9 \times 10^5}$

$= \underline{\underline{8.0 \times 10^{-19}}}$ C

(c) e = one electronic charge = 1.6×10^{-19} C

thus no. of charges = $\frac{Q}{e}$ = 5 a whole number.

8 $\frac{1}{2} m v^2 = Q V$

$v^2 = \frac{2QV}{m}$

$v = \sqrt{2\frac{Q}{m}V}$

$= \sqrt{2 \times 1.8 \times 10^{11} \times 250}$

$= \underline{\underline{9.5 \times 10^6}}\ \text{m s}^{-1}$

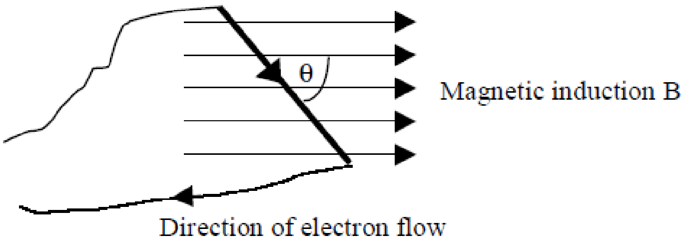
Notice that this speed is much less than the speed of light.

Tutorial 5.0

Electromagnetism

1. (a) A magnetic field is produced around moving charges.
   (b) The charge must be moving across the magnetic field, or the magnetic field must be changing relative to the position of the charge.
   (c) One tesla is the magnetic induction of a magnetic field in which a conductor of length one metre, carrying a current of one ampere perpendicular to the field, is acted on by a force of one newton.

2. (a) $F = IlB\sin\theta$
   (b) For a current in the direction shown, the force on the wire is directed into the page.



   (c) $F = IlB\sin\theta$ giving $9.5 \times 10^{-3} = 2 \times 25 \times 10^{-3} \times 0.70 \times \sin\theta$
   $\theta = 16^\circ$

3. $F = IlB\sin\theta$ $\quad 0.20 = 10 \times 0.8 \times B$ $\quad (\sin\theta = 1$ since θ is $90^\circ)$
   $B = 25$ mT $\quad (0.025$ T$)$

4. (a) $F = IlB\sin\theta$ $\quad 0.30 = I \times 0.50 \times 0.10$ $\quad$ for θ at 90° and $\sin\theta = 1$
   $I = 6$ A
   (b) If θ is less than 90°, $\sin\theta$ will be less than 1, and a larger current I will be required.

5. (a) $F = IlB\sin\theta$ $\quad F = 7.0 \times 200 \times 10^{-3} \times 0.15 \times \sin 35$ $\quad F = 0.12$ N
   (b) Out of the page, perpendicular to both the magnetic field and the wire.

6. Magnetic induction $B = \dfrac{\mu_0 I}{2\pi r}$

7. See Student Material page 20
   B is magnetic induction (T)
   μ_o is the permeability of free space (H m^{-1})
   I_1 and I_2 are the currents in the conductors (A) r is the distance between them (m)
   F/l is the force per unit length (N m^{-1})

8. $\dfrac{F}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$ giving $8.89 \times 10^{-6} = \dfrac{4\pi \times 10^{-7} \times 2.0 \times I_2}{2\pi \times 90 \times 10^{-3}}$ $\quad I_2 = 2.0$ A

9. Weight per m length = magnetic force per m length
   $0.075 = \dfrac{4\pi \times 10^{-7} \times I^2}{2\pi \times 5.0 \times 10^{-3}}$
   Current in each wire = 43 A

Tutorial 5.1

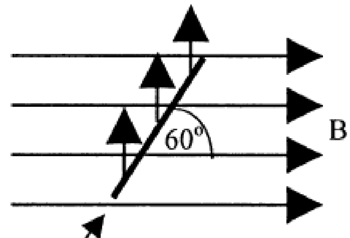
Force on a Conductor

1 (a) $F = Il\ B \sin\theta$

$= 7.5 \times 0.05 \times 0.04 \times \sin 60°$

$= \underline{\underline{0.013}}\ N$

(b)



(c) For maximum force, $\theta = 90°$ and all the conductor, 50 mm, is in the field.

2 $F = Il\ B \sin\theta$

$4.5 \times 10^{-3} = 1.4 \times 50 \times 10^{-3} \times 0.09 \times \sin\theta$

$\theta = \underline{\underline{46°}}$

3 (a) To remove tension in the supporting leads the magnetic force has to be equal and opposite to the weight of the wire.

$W = mg = 0.025 \times 9.8$

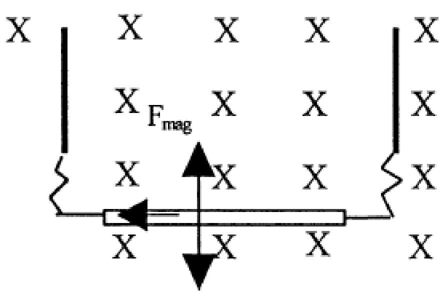
$= 0.245\ N$

using $F = I\,l\ B \sin\theta$ and $F = 0.245\ N$ and $\theta = 90°$

$$I = \frac{F}{l B \sin\theta} = \frac{0.245}{0.75 \times 0.50 \times 1}$$

$I = \underline{\underline{0.65}}\ A$

(b) Apply Right Hand Rule



4 (a) $$F = IlB\sin\theta$$

For $\theta = 90°$ the 0.25 m sides are perpendicular to the field

$$F = 0.25 \times 0.25 \times 0.40 \times 1$$
$$F = \underline{\underline{0.025}}\ \text{N}$$

(b) $T = Fr$ for **each** force $T = 0.025 \times 0.075$

Total torque $= 2 \times 0.025 \times 0.075 = 3.75 \times 10^{-3}$ N m (for each turn of wire)

Thus for the whole coil: Torque $= 120 \times 3.75 \times 10^{-3}$

$= \underline{\underline{0.45}}$ N m

(c) As the coil rotates the 0.25 m sides of the coil make angles less than 90° with the field. The force on the wire decreases so the torque decreases. When the coil is perpendicular to the field these sides are momentarily parallel to the field and the torque will be zero.

5 (a) Since the balance reading is less, this suggests that there is an **upward** force on the magnet assembly exerted by the current in the wire.

difference in readings = 95.6 g - 93.2 g = 2.4 g

$$F = 2.4 \times 10^{-3} \times 9.8 = 0.02352$$
$$= \underline{\underline{0.024}}\ \text{N}$$

(b) $$F = IlB\sin\theta \qquad (\theta = 90°)$$
$$0.024 = 4.0 \times 0.06 \times B \times 1$$
$$B = \underline{\underline{0.1}}\ \text{T}$$

(c) Reversing the direction of the current in the wire reverses the direction of the force. This direction is now downward and will **increase** the reading by 2.4 g.

new reading on balance = 95.6 + 2.4 = $\underline{\underline{98.0}}$ g

(d) When north faces north, the field is zero between the magnets (in centre). There will be no magnetic force on the wire. The balance will read 95.6 g.

6 (a) $$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

thus $$F = \frac{4\pi \times 10^{-7} \times 16 \times (2500)^2}{2\pi \times 0.20}$$
$$= \underline{\underline{100}}\ \text{N}$$

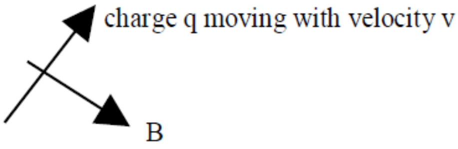
(b) If the wires were suspended freely they would attract each other. If they touched there would be a short circuit which could start a fire.

Tutorial 6.0

Motion in a magnetic field

1.

Consider a charge q moving with a constant speed v **perpendicular** to a magnetic field of magnetic induction B.



We know that $F = I\textit{l}B\sin\theta$.
Consider the charge q moving through a distance *l*. (The italic *l* is used to avoid confusion with the number one.)

Then time taken $t = \frac{l}{v}$ and current $I = \frac{q}{t} = \frac{qv}{l}$ giving $lI = qv$.

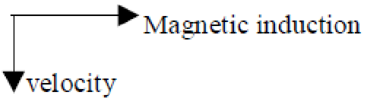
Substituting into $F = IlB\sin\theta$, with $\sin\theta = 1$ since $\theta = 90^{\circ}$, gives:

$$\boxed{F = qvB}$$

2. $F = qvB$ $\quad F = 1.6 \times 10^{-19} \times 3 \times 10^{4} \times 0.80 = 3.8 \times 10^{-15}$ N

3. (a) $F = qvB$ $\quad F = 1.6 \times 10^{-19} \times 2.0 \times 10^{5} \times 0.50 = 1.6 \times 10^{-14}$ N

(b) The force is perpendicular to both the velocity and the magnetic field.



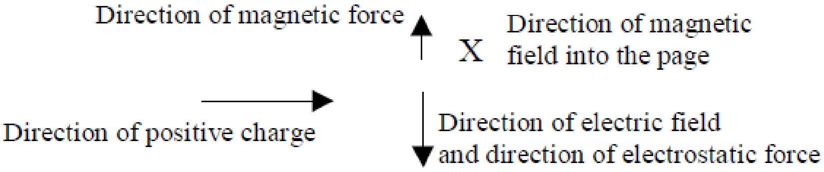
For these directions of v and B the force is vertically into the page.

(c) Central force $F = \frac{mv^2}{r}$ giving $qvB = \frac{mv^2}{r}$ and $r = \frac{mv}{qB}$

$$r = \frac{9.11 \times 10^{-31} \times 2 \times 10^{5}}{1.6 \times 10^{-19} \times 0.5} = 2.3 \times 10^{-6} \text{ m}$$

4. The direction of the velocity of the electron must make an angle with the direction of the magnetic field.
The component of velocity perpendicular to the field causes the electron to move in a circle.
The component of velocity parallel to the field causes the electron to move along the direction of the field.

5. (a) Electric and magnetic fields arranged so their forces on a charged particle oppose each other. The electric and magnetic field must be perpendicular to each other.

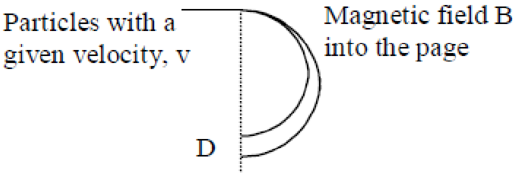


For the example shown above, a magnetic field is directed into the page and an electric field acts down the page. The magnetic force and the electrostatic force on a positive charge oppose each other. These fields are then said to be 'crossed'.
The fields are often adjusted such that the two forces are equal in magnitude, $F_e = F_m$. The velocity of the positive charge would then remain constant.

(b) $F_e = F_m$ also $F_e = Eq$ and $F_m = qvB$ giving $Eq = qvB$ and $v = \frac{E}{B}$.

(c) From the above equation the velocity does not depend on the mass or the charge of the particle but only on the values of E and B.

(d) Charged particles enter the region with the magnetic field only, at right angles to the field. The particle will move in a circle with a radius given by $r = \frac{mv}{qB}$.



The particles all have the same velocity hence the radius will depend on the mass and charge of each particle. Different ions will meet a detector at D in different places.

6. (a) From the question above $v = \frac{E}{B} = \frac{V}{Bd}$

(b) $qvB = \frac{mv^2}{r}$ or $evB = \frac{mv^2}{r}$ giving $\frac{e}{m} = \frac{v}{rB}$

(c) $\frac{e}{m} = \frac{v}{rB} = \frac{V}{rB^2 d}$

7. Velocity of each ion is given by $\frac{1}{2} mv^2 = qV$

giving $v^2 = \frac{2qV}{m}$ - - - - - - - - - - - - - {1}

radius of curved path $r = \frac{mv}{qB}$ - - - - - - - - - - - - - {2}

and $r^2 = \frac{m^2 v^2}{q^2 B^2}$ thus $r^2 = \frac{m^2}{q^2 B^2}\left(\frac{2qV}{m}\right)$ substituting for v^2

giving $r = \sqrt{\frac{2Vm}{qB^2}}$

The difference in the two radii $r_{13} - r_{12} = \sqrt{\frac{2V}{qB^2}}\left(\sqrt{m_{13}} - \sqrt{m_{12}}\right)$

$= 5 \times 10^{-3}$ m

Hence d = 0.01 m (Because of the subtraction above the answer is to one significant figure only.)

[Alternatively the velocity of each ion can be calculated using {1} above to give v_{13} and v_{12}. Then equation {2} used to give the difference in the radii $r_{13} - r_{12} = \frac{1}{qB}(m_{13}v_{13} - m_{12}v_{12})$. This involves more calculations and potential loss in accuracy.]

Tutorial 6.1

Charged Particles in Magnetic Fields

1 $F = q v B \sin\theta$

thus $B = \dfrac{F}{q v \sin\theta} = \dfrac{3.0 \times 10^{-17}}{1.6 \times 10^{-19} \times 1.0 \times 10^{7} \times \sin 45°}$

$= \underline{2.7 \times 10^{-5}}$ T

2 $F_m = q v B \sin\theta$ and $\theta = 90°$

$= 1.6 \times 10^{-19} \times 2.8 \times 10^{8} \times 3.3 \times 10^{-5} \times 1.0$

$= 1.478 \times 10^{-15}$ N

$F_g = m g$ notice the speed of proton is > 10% speed of light

Use $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ to find the relativistic mass m

$m = 4.660 \times 10^{-27}$

$F_g = 4.660 \times 10^{-27} \times 9.8 = 4.567 \times 10^{-26}$ N

thus $\dfrac{F_m}{F_g} = \dfrac{1.478 \times 10^{-15}}{4.567 \times 10^{-26}} = \underline{3.2 \times 10^{10}}$

If the relativistic mass is not used an answer of 9.0×10^{-10} is obtained!

3 $F = q v B = \dfrac{mv^2}{r}$ giving $r = \dfrac{m v}{q B}$

$\frac{1}{2} m v^2 = 4.2 \times 10^{-12}$ to find v from value given for E_k.

$v = \sqrt{\dfrac{2 \times 4.2 \times 10^{-12}}{1.673 \times 10^{-27}}} = 7.086 \times 10^{7}$ m s $^{-1}$

from above $r = \dfrac{m v}{q B} = \dfrac{1.673 \times 10^{-27} \times 7.086 \times 10^{7}}{1.6 \times 10^{-19} \times 0.28}$

$= \underline{2.6}$ m

4 from $qvB = \dfrac{mv^2}{r}$ $r = \dfrac{m v}{q B}$ and $v = r\omega = r \times 2\pi f$

giving $f = \dfrac{qB}{2\pi m}$ use this to compare frequency for α and electron

$\dfrac{f_\alpha}{f_e} = \dfrac{q_\alpha B}{2\pi m_\alpha} \times \dfrac{2\pi m_e}{q_e B} = \dfrac{q_\alpha m_e}{q_e m_\alpha} = \dfrac{2 m_e}{m_\alpha}$ since. $q_\alpha = 2 q_e$

$= \dfrac{2 \times 9.11 \times 10^{-31}}{6.68 \times 10^{-27}} = \underline{2.73 \times 10^{-4}}$

Alternatively: $f_e = 3.67 \times 10^{3} f_\alpha$

5 (a) The magnetic force supplies the central acceleration.

$qvB = \dfrac{mv^2}{r}$ giving $r = \dfrac{m v}{q B}$ also $v = r\omega$ and $\omega = 2\pi f$

hence $\dfrac{v}{r} = 2\pi f$ and $\dfrac{v}{r} = \dfrac{qB}{m}$ giving $f = \dfrac{qB}{2\pi m}$

(b) From the equation in (a) above: $B = \frac{2\pi mf}{q}$

$$B = \frac{2\pi \times 3.34 \times 10^{-27} \times 1.2 \times 10^{7}}{1.6 \times 10^{-19}} = 1.574$$

$= \underline{\underline{1.6}}\ T$

(c) At maximum radius R: $v = \frac{qBR}{m}$

E_k of deuterons emerging: $E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \times [\frac{qBR}{m}]^2$

$$E_k = \frac{q^2B^2R^2}{2m}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (1.574)^2 \times (0.50)^2}{2 \times 3.34 \times 10^{-27}}$$

$= \underline{\underline{2.4 \times 10}}^{-12}\ J$

6 (a) If undeflected:

$$F_{mag} = F_{elect}$$
$$q v B = q E$$
$$v = \frac{E}{B}$$

(b)

$$v = \frac{1.4 \times 10^5}{0.70}$$

$= \underline{\underline{2.0 \times 10}}^5\ m\ s^{-1}$

(c) (i) $r = \frac{m v}{q B}$ thus $m = \frac{q r B}{v}$

$$m = \frac{1.6 \times 10^{-19} \times 0.07 \times 0.7}{2.0 \times 10^5}$$

$= \underline{\underline{3.9 \times 10}}^{-26}\ kg$

(ii) The ions of the different isotopes will have different radii. They will therefore show up at different points on the photographic record. The less massive particles will have a larger radius.

7 (a) $v = \frac{q r B}{m}$ from $qvB = \frac{mv^2}{r}$

$$= \frac{3.2 \times 10^{-19} \times 0.45 \times 1.2}{6.68 \times 10^{-27}} = 2.59 \times 10^7$$

$= \underline{\underline{2.6 \times 10}}^7\ m\ s^{-1}$

(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.45}{2.59 \times 10^7}$$

$= \underline{\underline{1.1 \times 10}}^{-7}\ s$

(c) $E_k = \frac{1}{2} m v^2 = \frac{1}{2} \times 6.68 \times 10^{-27} \times (2.6 \times 10^7)^2$

$= \underline{\underline{2.3 \times 10}}^{-12}\ J$

8 (a) Particle X is moving in a direction parallel to the magnetic field. This means that it will not experience a magnetic force. Particle X will therefore carry on in a straight line with no change of speed.

Particle Y follows a circular path because it enters the magnetic field at right angles to the field direction.

Particle Z follows a helical (spiral) path because it enters the magnetic field at an angle.

(b) $r = \dfrac{m\,v}{q\,B}$ from $qvB = \dfrac{mv^2}{r}$

$$r = \frac{2 \times 10^{6} \times 1.673 \times 10^{-27}}{1.6 \times 10^{-19} \times 1.3 \times 10^{-5}}$$

$= 1.61 \times 10^3$ m

$= \underline{\underline{1.6}}$ km

9 (a) (i)

$F_{mag} = F_{elect}$

$q\,v\,B = q\,E$

thus $v = \dfrac{E}{B}$ and $E = \dfrac{V}{d}$

$v = \dfrac{V}{Bd}$

(ii) $B = \dfrac{9 \times 10^{-7}\,NI}{a}$

$= \dfrac{9 \times 10^{-7} \times 320 \times 0.31}{0.073} = 1.22 \times 10^{-3}$

$= \underline{\underline{1.2 \times 10^{-3}}}$ T

(iii) thus $v = \dfrac{V}{Bd} = \dfrac{1200}{1.22 \times 10^{-3} \times 0.045} = 2.19 \times 10^7$

$= \underline{\underline{2.2 \times 10^7}}$ m s^{-1}

(b) (i) $r = \dfrac{m\,v}{q\,B}$ from $qvB = \dfrac{mv^2}{r}$

and $\dfrac{e}{m} = \dfrac{v}{rB}$ since here $q = e$

(ii) $r = \dfrac{L^2 + y^2}{2y}$

$r = \dfrac{0.055^2 + 0.015^2}{2 \times 0.015}$

$= 0.108$ m

$$\frac{e}{m} = \frac{v}{rB} = \frac{2.19 \times 10^{7}}{0.108 \times 1.2 \times 10^{-3}}$$

$= \underline{\underline{1.7 \times 10^{11}}}$ C kg^{-1}

The accepted value for $\dfrac{e}{m}$ is 1.76×10^{11} C kg^{-1}

10 (a) $E = \frac{V}{d} = \frac{1200}{0.05} = \underline{2.40 \times 10^4}\ \text{V m}^{-1}$

(b) (i) $B = \frac{8\mu_0 NI}{\sqrt{125}\, r}$

$= \frac{8 \times 4\pi \times 10^{-7} \times 320 \times 0.25}{11.2 \times 0.068} = 1.058 \times 10^{-3}$

$= \underline{1.06 \times 10^{-3}}\ \text{T}$

(ii) $v = \frac{E}{B} = \frac{2.40 \times 10^4}{1.058 \times 10^{-3}}$

$v = \underline{2.27 \times 10^7}\ \text{m s}^{-1}$ (keeping 3 sig figs)

(c) (i) time taken to cross between plates $t = \frac{L}{v}$

deflection, $y = \frac{1}{2} a t^2 = \frac{1}{2} a \frac{L^2}{v^2}$

(ii) thus $y = \frac{1}{2} \times \frac{eE}{m} \times \frac{L^2}{v^2}$ since $a = \frac{F}{m} = \frac{eE}{m}$

$\frac{e}{m} = \frac{2yv^2}{EL^2} = \frac{2 \times 0.01 \times (2.27 \times 10^7)^2}{2.40 \times 10^4 \times 0.05^2}$

$= \underline{1.72 \times 10^{11}}\ \text{C kg}^{-1}$

11 (a) Electric potential energy = kinetic energy gained

$eV = \frac{1}{2} m v^2$ and $V = 1000$ V

thus $\frac{e}{m} = \frac{v^2}{2 \times 1000}$

(b) (i) $B = \frac{9 \times 10^{-7} NI}{r}$

$= \frac{9 \times 10^{-7} \times 320 \times 0.26}{0.068}$

$= \underline{1.10 \times 10^{-3}}\ \text{T}$

(ii) $v = \frac{E}{B} = \frac{2.0 \times 10^4}{1.10 \times 10^{-3}}$ since $E = \frac{1000}{0.05} = 2 \times 10^4$

$= \underline{1.82 \times 10^7}\ \text{m s}^{-1}$

(c) $\frac{e}{m} = \frac{v^2}{2 \times 1000} = \frac{(1.82 \times 10^7)^2}{2000}$

$= \underline{1.66 \times 10^{11}}\ \text{C kg}^{-1}$

(d) Accepted value for $\frac{e}{m} = 1.76 \times 10^{11}\ \text{C kg}^{-1}$

(i) An uncertainty of 5% in the value in (c) above:

gives $\frac{e}{m} = (1.66 \pm 0.08) \times 10^{11}\ \text{C kg}^{-1}$

(ii) The calculated uncertainty does not bring the measured value within range of the accepted value. The measured value is about 6% too low.

Tutorial 7.0

Capacitors in d.c. circuits

Numerical answers:

1. (a) $5{\cdot}0 \times 10^{-3}$ C
   (b) (i) 1·25 A
   (ii) current decreases exponentially

2. 0·5 μF

3. $2{\cdot}25 \times 10^{-5}$ C

4. (a) 1·0 μF
   (b) 0·8 μC

5. (b) 4·9 mF

6. (e) (i) 40 μA
   (ii) $4{\cdot}0 \times 10^{2}$ μC or $4{\cdot}0 \times 10^{-4}$ C

7. (e) 12 V

10. 8 ms

12. (b) (i) 8 s
   (ii) 7·3 V

Tutorial 7.1

Capacitors in a.c. circuits

Numerical answers:

2. (a) $3{\cdot}2 \times 10^3\ \Omega$
   (b) 1·6 mA

3. (a) $500\ \Omega$
   (b) 6·4 μF

4. (a) 0·24 A r.m.s
   (b) 4·8 μF

5. (a) 6·00 μF
   (b) 18·9 mA
   (c) 5·66 V

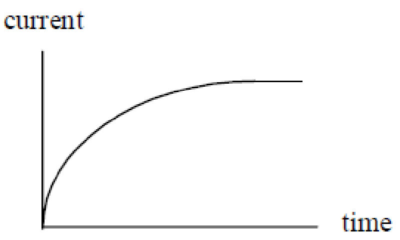
6. (a) $4\ \Omega$
   (b) $2{\cdot}7 \times 10^{-4}$ F (actual values $4\ \Omega$ and $3{\cdot}0 \times 10^{-4}$ F)

Tutorial 8.0

Self-inductance

1. (a) A magnet is moved in and out a coil. The coil is connected to a voltmeter and a deflection is observed when the magnet is moving relative to the coil, see Electrical Phenomena - Student Material page 27.
   (b) Increase the relative speed of the magnet and coil, increase the magnetic induction of the magnet, increase the number of turns on the coil.

2. (a)



   (b) When the switch is closed the current increases from zero. The magnetic field through the inductor will increase. An e.m.f. is induced across the inductor due to the changing magnetic field through the inductor. This induced e.m.f. acts against the current preventing the current reaching its maximum value immediately.
   (c) The current would reach its steady value quicker, see Electrical Phenomena – Student Material page 28.

3. When a steady current is passed through a coil a constant magnetic field is established through the coil. When the current through the coil changes, the magnetic field through the coil will change. A changing magnetic field will cause and induced e.m.f. through the coil.
   [**Note**: The induced e.m.f. will act in a direction to oppose the change causing it. Thus the induced e.m.f. produced when the current increases will act in a direction as to oppose the increase. It will act **against** the current direction.]

4. (a) When the current decreases the magnetic field will decrease and an e.m.f. will be induced.
   (b) The induced e.m.f. will act in the **same** direction as the current, that is it will try to keep the current steady and stop the change in the magnetic field. The energy needed to do this comes from the energy which was stored in the magnetic field. When the magnetic field decreases this energy is released and to conserve energy work has to be done.

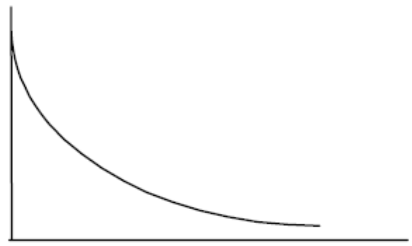
5. (a) $\mathcal{E} = -L\frac{dI}{dt}$
   (b) The unit of inductance is henry (H)
   (c) Energy $= \frac{1}{2} LI^2$

6. (a) As the current increases the magnetic field through the inductor increases. An e.m.f. is induced against the direction of the current. Thus the current takes time to reach its maximum value.
(b) Using V = I R steady current = 12/15 = 0.8 A
(c) Energy = ½ LI^2 = ½ x 0.40 x 0.8^2 = 0.13 J

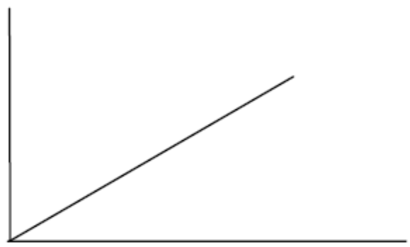
7. (a) p.d. = I R = 0.1 x 20 = 2 V
(b) induced e.m.f. = 8 – 2 = 6 V
(c) $\mathcal{E} = -L\frac{dI}{dt}$ 6 = -L x (-100)   L = 0.06 H
(d) Energy = ½ LI^2 = ½ x 0.06 x 0.10^2 = 0.30 mJ

8. None

9. (a) (i)



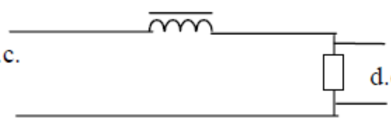
(ii)



(b) The reactance of the inductor is the opposition of the inductor to the alternating current. It is given by reactance $X_L = \frac{V}{I}$.

11. (a) When a switch in opened in a circuit containing an inductor the current will fall rapidly to zero. There will be a large change in the magnetic field through the inductor and this will cause a large induced e.m.f. at the switch terminals.

(b)



The inductor has a large opposition to a.c. signals. For d.c. signals the only opposition is the resistance of the coil. Assume that the resistance of the resistor is much larger than the resistance of the inductor. The p.d. across the inductor will be due to the a.c. signals and the p.d. across the resistor will be due to the d.c. signals. The inductor blocks the a.c.

Tutorial 8.1

Self-Inductance

1 (a) V across R = I R = 8.0 x 1.0 = 8.0 V

thus e.m.f. across inductor = 12 - 8.0 = 4.0 V

(b) $$\varepsilon = -L\frac{dI}{dt}$$

$$-4.0 = -L \times 400 \quad \text{(4.0 V is a back e.m.f)}$$

$$L = \frac{4.0}{400} = \underline{\underline{0.01}}\ \text{H}$$

(c) When the switch is closed and current is zero, all of the e.m.f. will be across the inductor: $\varepsilon = 12$ V.

$$\frac{dI}{dt} = \frac{\varepsilon}{L} = \frac{12}{0.01} = \underline{\underline{1200}}\ \text{A s}^{-1}$$

(d) $$I_{final} = \frac{\varepsilon}{R} = \frac{12}{1.0} = \underline{\underline{12}}\ \text{A}$$

(e) $$E = \frac{1}{2} L I^2$$

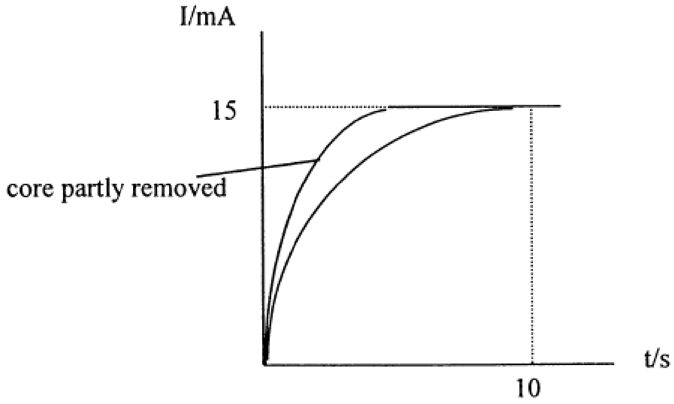
$$= \frac{1}{2} \times 0.01 \times 12^2$$

$$= \underline{\underline{0.72}}\ \text{J}$$

2 (a) (i) When the switch is closed the current starts to increase in the circuit. This *changing* current produces a *changing* magnetic field which in turn induces an e.m.f. across the coil. This e.m.f. opposes the build up of the current (Lenz's law). This is observed as a delay in the current reaching a final steady value.

(ii) When the current reaches its final steady value there is no induced e.m.f. across the inductor and therefore no back e.m.f. generated. The resistance in the circuit and the e.m.f. of the supply determine this steady current.

(b) The removal of the soft iron core reduces the inductance of the coil. This will result in a faster build up of current because the back e.m.f. will be less.



(c) $$R_{coil} = \frac{\text{e.m.f.}}{I_{final}} = \frac{3.0}{0.015} = \underline{\underline{200}}\ \Omega$$

3 (a) (i) p.d. across R at start is $\underline{\underline{0}}$ V

(ii) initial current is also $\underline{\underline{\text{zero}}}$.

(iii) initial induced e.m.f across L is the e.m.f. of the supply, $\underline{\underline{10}}$ V

(iv) $E = \frac{1}{2} L I^2$ but $I = 0$ A thus $E = \underline{\underline{0}}$ J

(b) (i) $V = IR = 0.04 \times 40 = \underline{\underline{1.6}}$ V

(ii) thus $\varepsilon = 10 - 1.6 = 8.4$ V

(iii) $\varepsilon = -L\frac{dI}{dt}$

$$-8.4 = -2.0 \times \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{8.4}{2.0} = \underline{\underline{4.2}} \text{ A s}^{-1}$$

(iv) $E = \frac{1}{2} L I^2$

$$= \frac{1}{2} \times 2.0 \times 0.04^2$$

$$= \underline{\underline{1.6 \times 10^{-3}}} \text{ J}$$

4 (a) The self-inductance of a coil is given by $\varepsilon = -L\frac{dI}{dt}$. The inductance is one henry if an e.m.f. of one volt is induced when the current changes at a rate of one ampere per second.

(b) (i) Lamp X lights more slowly due to the self-inductance of the inductor L. When the circuit is switched on the current grows and produces a changing magnetic field in the inductor. This in turn generates an e.m.f. which by Lenz's law opposes the original current. There is no such effect with a resistor, hence lamp Y lights immediately.

(ii) When the switch is closed $\varepsilon = 10$ V

$$\varepsilon = -L\frac{dI}{dt} \quad \text{and } L = \frac{10}{0.50} = \underline{\underline{20}} \text{ H}$$

(iii) Current in inductor branch $= \frac{P}{V} = \frac{3}{6} = 0.50$ A

p.d. across L $= 10 - 6.0 = 4.0$ V

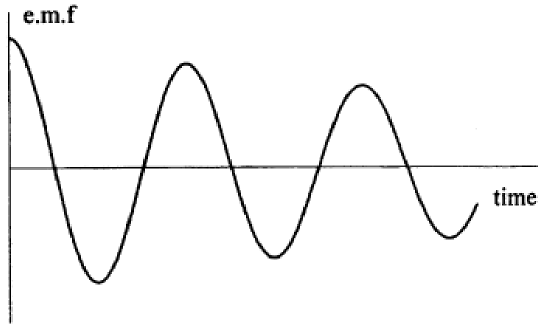
thus $R_L = \frac{V}{I} = \frac{4.0}{0.50} = \underline{\underline{8.0}}\ \Omega$ using V = IR

5 (a) When S is opened the current in the primary collapses. This produces a large change in magnetic field in the primary. This in turn produces a change in magnetic field in the linked secondary coil, which gives an large induced e.m.f. across the spark plug.

(b) If there are more turns in the secondary (step-up), a larger e.m.f. will be produced across the spark plug.

(c) The energy for the spark comes from the battery via the electromagnetic field set up in the coils and core.

6 (a)



The oscillations will be damped due to Lenz's law. The magnetic field in the coil will oppose the movement of the magnet.

(b) When the magnet momentarily stops the induced e.m.f. is zero. Relative movement is needed to induce an e.m.f.

(c) When magnet movement is reversed the induced e.m.f. will also be reversed.

(d) The fastest movement results in the maximum induced e.m.f.

Tutorial 8.2

Inductors and a.c.

Numerical answers:

2. (a) 250 Ω
   (b) 0·12 A

3. (a) 500 Ω
   (b) 1·6 H

4. (a) 0·12 A
   (b) 30 V

5. Lamp Z: no change in brightness when frequency is altered. The resistance of a resistor does not change with frequency.

   Lamp Y: as frequency increases the lamp dims because there is a greater back e.m.f. generated at a higher frequency. The inductive reactance has increased, i.e. the opposition to a.c. increases.

   Lamp X: as frequency increases the lamp becomes brighter because the capacitor allows a greater current. to pass. The capacitive reactance has decreased, i.e. the opposition to a.c. decreases.

6. (a) Loudspeaker A will reproduce the high frequency signals while loudspeaker B will reproduce the low frequency signals.

   (b) Both high and low frequency signals have a choice of path at the top of the circuit, where the inductor L and capacitor C_2 join. Higher frequency signals will pass through the capacitor, C_2, because there is less opposition (capacitive reactance) for that route. After passing loudspeaker A the high frequency signals take the low opposition route through capacitor C_1.

   The low frequency signals will pass through the inductor because this route has a lower opposition (inductive reactance) for low frequencies. The low frequencies signals, after passing through inductor L, will pass through loudspeaker B rather than pass through C_1. A capacitor has a larger opposition to low frequency signals.

7. (b) (i) $6·4 \times 10^{-6}$ F
   (ii) 2·6 H (actual values $6·0 \times 10^{-6}$ F and 2·6 H)

Tutorial 9.0

Electromagnetic radiation

Numerical answers:

4. (a) $4{\cdot}0 \times 10^{2}$ V m^{-1}
   (b) $4{\cdot}5 \times 10^{14}$ Hz
   (c) (ii) $1{\cdot}3 \times 10^{-6}$ T